Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Administration Manual for Supervising Teachers

June 2004

Manitoba
Building for the Future

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Manitoba Education, Citizenship and Youth,
Assessment and Evaluation Branch,
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Preface

This *Administration Manual for Supervising Teachers* is distributed to teachers and administrators, and must be read prior to administering the standards test. The rules and guidelines in this manual must be strictly observed to ensure the consistency of standards test administration.

Please use this manual in conjunction with the registration package, the *Policies and Procedures for Standards Tests* handbook and the *Information Bulletin* (these documents are available on our website at: http://www.edu.gov.mb.ca/ks4/assess/publications.html) to gain a full understanding of the protocols and procedures associated with this test. This manual contains general information about the test, step-by-step instructions for the test administration, and policies regarding absenteeism, lateness, violation of rules, etc.

If you encounter exceptional circumstances not addressed in this manual, or if you have further inquiries, contact Carol Chabbert, Administrative Officer, at 945-0747 or toll-free at 1 800 282-8069, extension 0747.

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General Information

Manitoba Education, Citizenship and Youth develops all standards tests in collaboration with teachers from across the province. This standards test is based on *Senior 4 Mathematics Manitoba Curriculum Framework of Outcomes and Senior 4 Standards (2001).*

The standards test is locally marked and counts for 30% of each student's final grade in the course. Student results on the test will be released to students according to local policy on the date designated by local jurisdictions.

All students enrolled in Senior 4 Pre-Calculus Mathematics (40S), except those who have been exempted, must write the Senior 4 Pre-Calculus Mathematics (40S) Standards Test on June 8, 2004. However, a student may rewrite a mandatory provincial standards test at any regularly scheduled administration without being registered in the course (see *Policies and Procedures for Standards Tests*, Section 3.1).

Marking instructions are provided by the department immediately following test administration. School principals will receive the *Marking Guides* for the Senior 4 Pre-Calculus Mathematics (40S) Standards Test by June 8, 2004. The *Marking Guides* are to be stored in a secure location until 12:30 p.m. on June 8, 2004.

Students are expected to write the standards test as scheduled near the end of the semester in which they take the course (e.g., January for first-semester courses, June for second-semester and full-year courses).

All schools should receive the test materials by May 31, 2004. The number of test booklets sent to each school is based on the February 20, 2004 registration information provided by the school. Please advise Carol Chabbert, Administrative Officer, at 945-0747 or toll-free at 1 800 282-8069, extension 0747, immediately if the number of students registered to write the test has increased since registration.

Manitoba Education, Citizenship and Youth supports local marking by:

- training representatives from local jurisdictions to act as local marking co-ordinators;
- collecting and scanning all *Answer/Scoring Sheets* from jurisdictions to provide them with test result summaries;
- centrally marking a random sample of locally-marked test booklets to provide feedback to jurisdictions on their local marking; and
- providing provincially-funded local jurisdictions with $10.00 for each test administered in accordance with the test schedule noted on the following page and marked locally. The calculation is based on the number of students whose completed marking scan sheets have been received.

Features of the test

The Senior 4 Pre-Calculus Mathematics (40S) Standards Test consists of two parts:

- Part 1—open-response questions where a scientific or graphing calculator may be used; and
- Part 2—selected-response, restricted-response and open-response questions where a calculator cannot be used.

Test schedule

The following table summarizes the schedule for the Senior 4 Pre-Calculus Mathematics (40S) Standards Test:

Date	Session	Preparation Time	Activity	Time Required for Test
Tuesday, June 8, 2004	1	15 minutes preparation and instructions	Part 1	60 minutes
			Part 2	120 minutes

The test must be written between 8:30 a.m. and 12:30 p.m. on Tuesday, June 8, 2004.

Policies for Standards Tests

Principals and supervising teachers should be aware that the *Policies and Procedures for Standards Tests* handbook has been distributed to all schools and is also available on our website at: http://www.edu.gov.mb.ca/ks4/assess/publications.html. This handbook outlines current policies and procedures related to the administration of standards tests and reflects the principles and guidelines contained in *Principles for Fair Student Assessment Practices for Education in Canada (1993).*

Adaptations

Schools are asked to identify and submit requests for test adaptations **at the time of student registration**. These requests should be submitted on the *Request for Adaptations Form* included with the registration package. Following departmental review of the adaptation requests received, schools will be advised of the department's decision regarding these requests (see *Policies and Procedures for Standards Tests*, Sections 4.0, 4.1 and 4.2).

Exemptions

Decisions for student exemptions that fall within the guidelines set out in *Policies and Procedures for Standards Tests* (Sections 5.0, 5.1 and 5.2) are made at the school level and do not require departmental approval. Schools are required, however, to report school-based exemptions on the *Student Exemption Report* included with the registration package. If additional student exemptions are needed after the registration deadline date, an additional *Student Exemption Report* should be submitted as soon as possible.

For exemptions that fall outside the parameters of policy and procedure guidelines, requests should be made **at the time of student registration**. Please submit requests in writing to Wenda Dickens, Acting Director, Assessment and Evaluation Branch, 71–1567 Dublin Avenue, Winnipeg, Manitoba R3E 3J5.

Re-read provisions

As the reporting of individual student scores on locally marked standards tests is the responsibility of each local jurisdiction, the re-read policy for these tests is determined locally. Should a re-read be required for a test booklet that has been sent to the department as a part of the feedback on the marking process, the department will return a photocopy of the test booklet upon request. Please contact Carol Chabbert, Administrative Officer, at 945-0747 or toll-free at 1 800 282-8069, extension 0747.

Return of tests to students

Local jurisdictions decide whether to keep the test booklets or return them to students. The department will return the random sample of test booklets to schools within three months of marking.

Test confidentiality

Test materials must be kept secure and sealed until administration.

Test booklets must not be reproduced nor information from them communicated in any way until test administration is completed.

The *Marking Guides* are to be stored in a secure location until the test administration is complete.

Student materials

Students may use the following materials as specified, during the Senior 4 Pre-Calculus Mathematics (40S) Standards Test:

- HB pencil;
- calculator (scientific or graphing—**Part 1 only**);
- eraser; and
- ruler.

Students are not permitted to exchange materials during the test.

The following materials must **not** be used during the test:

- classroom notes, textbooks and other such materials;
- subject-related materials;
- dictionaries or translation resources;
- any electronic devices such as phones, pagers or computers; and
- extra paper (additional blank pages for student use are included at the end of the test booklets)

Students leaving the room

Students are expected to remain in the room for Part 1 (60 minutes) and until they have completed Part 2 of the test.

However, any student who must leave and re-enter the room should be accompanied by a replacement supervisor. This practice ensures that the supervising teacher can remain in the test room with the students at all times.

Late arrivals

Students who arrive more than one hour after the test has started are not allowed to write the test. Students who arrive late but within the first hour of the test are permitted to write but no extra time is allowed. The exception will be if students arrive late for reasons beyond their control. Under these circumstances, the local jurisdiction may allow extra time to write the test. The test

however, may not start after 9:30 a.m. Please note that late students who are granted extra time may only use their calculators for Part 1 of the test (maximum of 60 minutes). The classroom teacher should record on the *Supervising Teacher Report* the booklet numbers of students who arrived late and provide details of the circumstances which caused the student(s) to arrive late.

Student absences

Standards tests cannot be written at a later date and reported as a standards test result. Therefore, schools have the responsibility to provide a final mark to absent students. The final mark for these students must be identified as a school-based assessment and evaluation practice only (see *Policies and Procedures for Standards Tests*, Sections 6.0, 6.1 and 6.2 for additional information).

Students who formally refuse to write a standards test (by their own or by parental choice) or who are absent without a legitimate reason will receive a mark of 0% on the test.

Responding to student questions

Avoid making any directing/guiding comments during the test, other than those required for distribution of materials and instructions to students.

No explanations, translations, rephrasing, clarifying statements or comments of a similar nature may be provided about the test content unless otherwise specified. Supervising teachers can, however, provide clarification regarding test taking rules or procedures.

If a student asks a question related to test content, a suggested response is, "I'm sorry. I can't answer your question. Do your best with it."

Suspected errors in printing or wording should not be communicated to students unless specific instructions are received from the Assessment and Evaluation Branch. You may however contact Chantal Bérard, Assessment Consultant, at 945-7590 or toll-free at 1 800 282-8069, extension 7590 to report any suspected errors.

Violation of rules

The *Test Taking Rules* are read aloud to students by the supervising teacher immediately before the test (see Appendix 1, page 17).

Document any violation of rules on the *Supervising Teacher Report*. This documentation should include:

- the test booklet number; and
- a description of the circumstances, including available evidence.

The student may be permitted to complete the test.

The consequences of any violation of rules is a local decision.

Damaged or misprinted materials

Students should be assured that they will not be penalized for any obvious error in printing or wording. Local marking co-ordinators will be advised of any known errors before the marking session. Return the unused damaged or misprinted booklet(s) with the reports and forms being sent to the department immediately after the test session.

On test administration date

If a damaged or misprinted test booklet is discovered **at the start** of the test session:

- replace the damaged or misprinted test booklet with a new test booklet; and
- print "DAMAGED" on the test booklet cover and on the *Attendance Form*.

During test administration

If a damaged or misprinted test booklet is discovered **during** the writing of the test:

- give the student a new test booklet (the student keeps the damaged or misprinted test booklet until the end of the session);
- instruct the student to continue responding to test items in the new test booklet;
- collect both test booklets at the end of the test session and ensure that both booklets are kept together by bundling them with an elastic band;
- cross out the existing booklet number on the new test booklet and change it to the original booklet number so that **all student responses are recorded under the original booklet number**;
- indicate on the *Attendance Form* that two booklets have been used for one student (see Appendix 4, page 20); and
- record the damaged or misprinted test booklet number on the *Supervising Teacher Report*.

Responsibilities of School Principals

It is the responsibility of the principals to ensure that the tests are kept confidential until test administration is completed.

Other responsibilities are listed below.

If tests are not in your school by May 31, 2004, contact Carol Chabbert, Administrative Officer, at 945-0747 or toll-free at 1 800 282-8069, extension 0747.

Use the *Packing List* to confirm that all materials have been received. Count the shrink-wrapped test booklets to ensure that there are enough booklets for the number of students writing the test. Keep the shrink-wrapped test packages unopened until the morning of the test. If additional test materials are needed, immediately contact Carol Chabbert, Administrative Officer.

Store test materials in a secure area in the school.

Provide supervising teachers with a listing of MET numbers for the students registered to write the test. Call the Professional Certification and Student Records Unit at 1 800 667-2378 to obtain MET numbers for new students.

Make arrangements to meet the requirements of the adaptations approved for your students (see Adaptations, page 3 and *Policies and Procedures for Standards Tests*, Sections 4.0, 4.1 and 4.2).

Co-ordinate test scheduling and space requirements.

Minimize the frequency of school announcements during the test session.

Ensure that a replacement supervisor is available to accompany students who must temporarily leave the room.

Provide the test materials to the supervising teacher on the morning of the test.

Complete the *Principal Feedback Form* before the end of the test session.

Advise the Assessment and Evaluation Branch of school closures or suspended school bus transportation services that occur on the test date as a result of unusual or extenuating circumstances or inclement weather conditions. Please contact Chantal Bérard, Assessment Consultant, at 945-7590 or toll-free at 1 800 282-8069, extension 7590.

Responsibilities of Supervising Teachers

Supervising teachers administer the test using the step-by-step procedures outlined in the following section.

(✓)	**Advance preparations**
	1. Check with your principal to ensure that:
	– appropriate scheduling and allocation of space for the test has occurred;
	– the test materials itemized on the *Packing List* have been inventoried;
	– a replacement supervisor is available to accompany students who must temporarily leave the room during the test;
	– arrangements have been made to accommodate the adaptations approved for students; and
	– a list of MET numbers for students writing the test is available.
	2. Ensure that all subject-related materials have been removed from the walls and boards.
	3. Notify students in advance of the:
	– time, location and duration of the test session;
	– materials needed to write the test;
	– *Test Taking Rules* (see Appendix 1, page 17);
	– minimum time a student must remain in the test room (60 minutes for Part 1 and until completion of Part 2).
	4. Ensure that you have extra supplies and that there is a functioning pencil sharpener and stapler in the room. Students must not exchange materials during the test.
	5. If there is more than one teacher supervising the test, photocopy the *Supervising Teacher Report*.
	6. Review the sample forms in the Appendices.
	7. Please note that the pre-printed *Demographic Data Forms* already have the students' names and MET numbers and should be distributed accordingly.
(✓)	**Prior to test session**
	1. Ensure that you have access to the following test materials:
	– *Packing List*;
	– *Part 1* and *Part 2* test booklets;
	– *Attendance Forms* (enclosed with shrink-wrapped test booklets);
	– *Student Registration Update Form*;
	– extra copies of test booklets (four extras);

	– *Demographic Data Forms* (pre-printed and generic);
	– a list of MET numbers for students writing the test;
	– *Answer/Scoring Sheets*;
	– *Supervising Teacher Report*;
	– *Supervising Teacher Feedback Form*;
	– *Principal Feedback Form* (this must be given to the principal and collected before returning materials);
	– *Administration Manual for Supervising Teachers* (this document); and
	– labelled envelope for return of materials to the Assessment and Evaluation Branch.
	2. Count the number of *Part 1* and *Part 2* test booklets. The number of test booklets should equal the number requested by the school plus the additional 4 booklets. If you need to split a bundle of booklets between classrooms, make a photocopy of the *Attendance Form*. Adjust each copy of the *Attendance Form* to reflect the booklets being used in each classroom.
	3. Arrange the desks so they are spaced as far apart as possible.
	4. Remove the following from the envelope labelled "For the Return of Demographic Data Forms":
	– pre-printed *Demographic Data Forms* in alphabetical order for every student registered to write the test. Ensure that the school name, school code, and MET numbers appearing on the pre-printed forms are accurate (see Appendix 2, page 18); and
	– generic *Demographic Data Forms* for students who joined the classroom after registration, and who are scheduled to write the test (see Appendix 3, page 19). **Note:** Only **one** *Demographic Data Form* should be completed for each student. If a pre-printed *Demographic Data Form* exists for a student, that is the form that should be used. **DO NOT** complete a generic *Demographic Data Form* for that student.
	5. Sort the pre-printed *Demographic Data Forms* for distribution.
	6. Make arrangements per school policy to ensure that students who are exempted from writing the test are accommodated in an alternate setting.
(✓)	**Beginning the test session**
	1. Distribute:
	– *Part 1* test booklets. **Note:** *Part 2* test booklets and *Answer/Scoring Sheets* are distributed after 60 minutes; and
	– *Demographic Data Forms*.
	2. Tell students that while they are working you will be circulating the *Attendance Form* so they can print their names beside their test booklet number.

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	3. Read aloud the instructions for completing *Demographic Data Forms*. The instructions are found on the form itself. Direct students to print and shade in all the information requested. Remind students to accurately record the program (English, French Immersion, or Français) in which they are enrolled.
	4. Read aloud the *Test Taking Rules* to students (Appendix 1, page 17).
	5. Read aloud the *General Directions* to students (inside the front cover of the *Part 1* test booklet).
	6. Remind students to:
	– pace themselves in writing the test;
	– use the *Answer/Scoring Sheet*, which will be distributed with the *Part 2* test booklet, for responding to selected-response questions; and
	– check over their work to ensure that all questions have been attempted.
	7. **Instruct students to turn the page and begin. Record the start time for Part 1 on the *Supervising Teacher Report*.**
	8. Collect the *Demographic Data Forms*.
(✓)	**During the test session**
	1. Supervise the completion of the *Attendance Form* by ensuring that students print their name beside their test booklet number (see sample *Attendance Form*, Appendix 4, page 20). **Note:** This is the only record connecting students' identities to their *Part 1* and *Part 2* test booklets.
	2. To prepare for the distribution of Part 2 of the test, match-up each *Part 2* test booklet with the appropriate numbered *Answer/Scoring Sheet*. You may wish to insert the *Answer/Scoring Sheets* inside the front cover of the appropriate test booklet.
	3. Verify that all the information on each *Demographic Data Form* is accurately printed and shaded. Place all used forms in numerical order by booklet number followed by any unused forms in the labelled envelope provided. Do not use elastic bands, staples or paper clips on these forms as they may damage the sheets. Do not seal the envelope.
	4. Half-way through Part 1 of the test session (after 30 minutes), advise students of the amount of time remaining for Part 1 by writing it on the board.
	5. After 60 minutes, ask the students to place their calculators on the floor for the remainder of the test session. Students retain their *Part 1* test booklet for the entire test session. Advise students they may continue working on *Part 1* but they may not use their calculators once *Part 2* is distributed.
	6. Distribute the *Part 2* test booklet and the *Answer/Scoring Sheet*, but **before** doing so please note: **The booklet numbers for *Part 1* and *Part 2* and the number on the *Answer/Scoring Sheet* need to be identical for each student. Advise students that as they receive the *Part 2* test booklet and an *Answer/Scoring Sheet* to verify that these items have the same number as their *Part 1* test booklet. Tell students that their answers to the selected-response questions are to be shaded on the back of the *Answer/Scoring Sheet*.**

	7. **Instruct students to turn the page and begin. Record the start time for Part 2 on the *Supervising Teacher Report*.**
	8. Half-way through Part 2 of the test session, advise students of the amount of time remaining by writing it on the board.
(✓)	**Ending the test session**
	1. Have the students place their *Answer/Scoring Sheets* inside the front cover of the *Part 1* test booklet. Collect both test booklets from each student.
	2. As students submit their test materials, ensure that each test booklet number matches the information on the *Answer/Scoring Sheet* and the *Attendance Form* (e.g., cross-reference student name and test booklet number). Each student must have the same booklet number on all test materials.
	3. Place a check "✓" under Part 1 and Part 2 of the *Attendance Form* in the "Booklets Submitted to LMC for Marking" columns (see Appendix 4, page 20).
	4. **Record the time students finish writing the test on the *Supervising Teacher Report*.** Do not include extra time given for late arrivals or adaptations.
	5. Stack the test booklets in numerical order in 1 pile.
	6. If marking is conducted in the classroom or at the school, leave the *Answer/Scoring Sheets* inside the front cover of each *Part 1* test booklet. After marking, place all used *Answer/Scoring Sheets* in numerical order followed by any unused forms. Place the forms in the labelled envelope. Do not seal the envelope. If marking is conducted in a central location, place all used *Answer/Scoring Sheets* in numerical order followed by any unused forms. Place the forms in the labelled envelope. Do not seal the envelope.
	7. Complete the *Supervising Teacher Report*. Note any unusual circumstances on this report.
	8. Record and shade in the appropriate Reason Code on the pre-printed *Demographic Data Form* for all students who were absent for the session (see Appendix 2, page 18).
	9. Record and shade in the appropriate Drop-in Code on the generic *Demographic Data Form* for those students who were not included at the time of registration but were present to write the test (see Appendix 3, page 19).
	10. Ensure that the number of check marks in the "Booklets Submitted to LMC for Marking" column on the *Attendance Forms* matches your count of the booklets and *Answer/Scoring Sheets* for submission.
	11. Complete and check the *Student Registration Update Form* if applicable. Use the reason codes (Appendix 5, page 21) provided.
	12. Make two copies of the *Attendance Forms* and *Supervising Teacher Report*. One copy must be retained by the school and one forwarded to the local marking site, if a centralized marking model is used.
	13. Collect the *Principal Feedback Form*.

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	14. Complete the *Supervising Teacher Feedback Form.*
	15. Return all test booklets and other test-related materials to the secure area in the school until commencement of marking or until materials are to be forwarded to the local marking site.
	16. Send materials according to the instructions on the following pages.

Points to Remember

- Remain in the classroom throughout the entire test session and engage in work solely related to the duties of supervising the test.
- A replacement supervisor must be available to assist you.
- Store all documents not in use in a secure area in the school.
- Ensure that once a booklet number has been assigned to a student, all materials distributed to that student have the same booklet number.
- Students **may** review or complete work from Part 1 during the Part 2 portion of the test session. Students **cannot**, however, use their calculator during Part 2 of the session.
- Students **may not** bring anything produced outside of the test room.

Submitting materials on June 8, 2004 (the end of the test)

It is essential that the following materials are packaged and forwarded to the local marking site immediately after completion of the test if a centralized model is used.

- **All used** test booklets;
- **All used** *Answer/Scoring Sheets*;
- Copy of the *Attendance Forms*; and
- Copy of the *Supervising Teacher Report*.

All other documentation should be verified and returned to the Assessment and Evaluation Branch after the test session.

- *Attendance Forms*;
- **All used** *Demographic Data Forms* (in envelope provided);
- *Student Registration Update Form*;
- *Supervising Teacher Report*;
- *Supervising Teacher Feedback Form*;
- *Principal Feedback Form*; and
- **All** damaged or misprinted booklets.

Schools in the City of Winnipeg return their envelopes to the Assessment and Evaluation Branch by calling Mid-Canada Courier (989-5630) and arranging pick-up between **1:00 p.m. and 4:00 p.m.** on **June 8, 2004**. The person placing the call should quote **account number E901**, provide the school name, address and the name of the contact person.

Schools outside the City of Winnipeg return their envelopes to the Assessment and Evaluation Branch by using the Xpresspost Flexipack envelope provided and taking it to the post office.

Submitting materials after local marking

(to be received by the Assessment and Evaluation Branch no later than June 18, 2004)

When local marking is done at a central location, the **local marking co-ordinator** is responsible for sending the materials to the Assessment and Evaluation Branch. When local marking is done at the classroom level, the **teacher** is responsible for sending the materials to the Assessment and Evaluation Branch.

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After local marking has been completed, prepare a package to be received by the Assessment and Evaluation Branch no later than June 18, 2004. This will ensure there is sufficient time for processing prior to semester end.

The package must include:

- **All** *Answer/Scoring Sheets*;
- Test booklets requested by the department for feedback on the marking process;
- *Sample Request List*; and
- *Local Marking Co-ordinator Feedback Form*.

Jurisdictions within the City of Winnipeg send packages directly to the Assessment and Evaluation Branch via **Mid-Canada Courier (989-5630)**. The person placing the call should quote **account number E901**, give the jurisdiction name, address, and the name of the contact person.

Jurisdictions outside the City of Winnipeg return their package by Xpresspost. It is important to package all materials in the **Xpresspost Flexipack envelope** provided whenever possible.

After the package is sealed, deliver it to the local post office.

We have provided sufficient materials for the return of **one** envelope/package. If you must send a second envelope/package, photocopy the mailing address label and affix the copy to the second package. Note you must use a photocopy of the mailing address label as it provides Canada Post with billing information. Ask the postal clerk for an additional bar code label to attach to the second envelope/package. Ensure that the school representative retains the bar code number in case tracing the package is required.

Return **all** *Answer/Scoring Sheets*. We remind you that payments will be made according to the number of completed marking sheets received.

Appendices

Appendix 1: Test Taking Rules

Senior 4 Pre-Calculus Mathematics (40S) Standards Test

The supervising teacher is expected to read these rules aloud to students at the beginning of the Senior 4 Pre-Calculus Mathematics (40S) Standards Test:

- Check the title of your test booklets carefully.
- During this test session, do not proceed until instructed to do so.
- If you receive a damaged or misprinted booklet, raise your hand and I will give you a new one.
- You are expected to remain in the test room for the allotted time for Part 1 and until you complete Part 2. (You may leave before that time in exceptional circumstances such as illness, when accompanied by a supervisor.)
- At the beginning of the session you should only have a *Part 1* test booklet, a *Demographic Data Form*, an HB pencil, an eraser, a ruler and your calculator (scientific or graphing, for Part 1 only). No other materials are permitted. Do not exchange test materials. When the *Part 2* test booklet is distributed, you will also receive an *Answer/Scoring Sheet* to record your answers to the selected-response questions. When I hand out the *Part 2* test booklet make sure the numbers on the *Part 1* and *Part 2* test booklets match the number on your *Answer/Scoring Sheet*.
- Your calculator must remain on the floor throughout the administration of Part 2. You may keep your *Part 1* test booklet for the entire session and continue working on Part 1 if you wish, **but** you may not use your calculator.
- You will not give assistance to, nor receive assistance from another student. If you require something during the test, raise your hand and I will come to you.
- During the test I can help you with the directions, but I will not be able to help you with the test questions.
- Do **not** identify yourself, your school, teacher or town in your test booklet.
- When I ask you to stop writing, put your pencil down immediately.
- You may not discard any materials. The test booklet must remain intact unless otherwise instructed. You may **not** leave the room with any materials pertaining to the test.

Appendix 2: Sample Pre-printed Demographic Data Form

DEMOGRAPHIC DATA FORM / FORMULAIRE DE DONNÉES DÉMOGRAPHIQUES

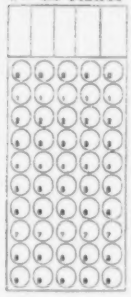
SENIOR 4 PRE-CALCULUS MATHEMATICS STANDARDS TEST – JUNE 2004 /
TEST BASÉ SUR LES NORMES MATHÉMATIQUES : PRÉ-CALCUL DE SECONDAIRE 4 – JUIN 2004 1

School/ École	9999 SCHOOL NAME	Program/ Programme	ENGLISH
Name/ Nom	SURNAME, GIVEN		
Birth Date/ Date de naissance	01/01/01	MET #/ N° MET	999-999-999

INSTRUCTIONS TO THE STUDENT:

1. Write and shade in the **Booklet No.**
2. Use an HB pencil only. Do not use ink, ball point, or felt tip pens.
3. Erase cleanly any marks you wish to change.
4. Do not mark other areas on this form.
5. If the pre-printed information is inaccurate, please cross it out and write in the correct information.

Booklet No./ N° de cahier



OR/OU

DIRECTIVES À L'ÉLÈVE :

1. Inscrire votre **N° de cahier** et noircir les cercles qui y correspondent.
2. Utiliser seulement un crayon HB. Ne pas utiliser de stylo à encre ou à bille, ni de crayon feutre.
3. Bien effacer toute marque que vous désirez changer.
4. Ne pas faire de marques à d'autres endroits sur ce formulaire.
5. Si les renseignements pré-imprimés sont inexacts, les rayer et écrire les renseignements exacts.

FOR TEACHER USE ONLY:

Write and shade in the **Reason Code** if the student is absent for **all** test sessions. Please see list of reason codes and instructions enclosed.

Reason Code/ Code désigné

Other/Autre :

À L'USAGE DE L'ENSEIGNANT SEULEMENT :

Écrire le **Code désigné** et noircir les cercles qui y correspondent si l'élève est absent pour **toutes** les séances de test. Veuillez vous reporter à la liste de codes et aux directives ci-incluses.

9999

Appendix 3: Sample Generic Demographic Data Form

Appendix 3: Sample Generic Demographic Data Form

DEMOGRAPHIC DATA FORM

Manitoba

NAME (Last)

NAME (First)

INSTRUCTIONS

- Use an HB pencil only.
- Do not use ink, ballpoint, or felt tip pens.
- Shade the circle completely.
- Erase cleanly any marks you wish to change.
- Do not mark other areas on this form.

EXAMPLES

INCORRECT	CORRECT
A ✓ C D	A B C ●
✗ B C D	
A B ● D	
A B C ↺	

PROGRAM INFORMATION

E English Program

I French Immersion Program

F Français Program

PROVINCIAL STUDENT (MET) NO.

BIRTH DATE

MONTH: Jan, Feb, Mar, Apr, May, June, July, Aug, Sept, Oct, Nov, Dec

DAY

YEAR

BOOKLET NO.

SCHOOL CODE

REASON CODE

NOTE TO TEACHER
Complete the Reason Code only for students who dropped in to the school and are present to write the test.

Appendix 4: Sample Attendance Form

School Number: 9999
Somewhere School
Somewhere Address
Somewhere MB R0R 0R0

Page _____ of _____

ATTENDANCE FORM—JUNE 2004

SENIOR 4 PRE-CALCULUS MATHEMATICS (40S) STANDARDS TEST

Booklet Number	Student Name (*please print*)	Booklets Submitted to LMC* for Marking (*check* ✓)	
		Part 1	Part 2
30001	Karen Masi	✓	went home sick
30002	Walter Woronowski	✓	✓
30003	Michael Smith	✓	✓
30004	DAMAGED		
30005	Jason Chu	✓	✓
30006	Shelley Bracken (also 30008)	✓	✓
30007	Bonnie Walters	✓	✓
30008	Used with 30006 (damaged)		
30009	UNUSED		
30010	UNUSED		

Note: Record any absences on the *Student Registration Update Form*.

Mr. John Ormiston
Supervising Teacher's Name (*please print*)

Please retain a copy of this form for your records. You will need a copy of this form to link your locally marked test with a student's name.

***Local marking co-ordinator**

Appendix 5: List of Reason Codes to Record Changes in Standards Test Student Registration Numbers

(Please use these codes to complete the *Student Registration Update Form* and the *Demographic Data Form* when returning to the Assessment and Evaluation Branch in June 2004)

ABSENT	
01	arrived too late to write the test
02	did not write due to weather
03	family/personal reasons
04	medical/illness/accident
05	out-of-town activity (athletic/academic)
06	religious objections
07	student refused to write
08	unexcused
09	working
10	suspended
19	other (please specify)

WITHDRAWN	
22	dropped course
23	incarcerated/court
24	medical/illness/accident
25	family/personal reasons
26	poor attendance/progress
27	quit school
28	expelled
29	other (please specify)

ERRORS IN REGISTRATION	
20	changed program
21	deceased
50	already has credit
51	auditing the course
52	exchange student
53	registered twice
54	quarter system projected registration
55	registered in wrong grade/course
59	other (please specify)

INCOMPLETE	
40	continuing course/program next semester
41	no work completed
49	other (please specify)

EXEMPTIONS	
60	emotional/psychological condition or circumstances
61	"M" credit designation
62	"E" credit designation
63	"phase d'accueil—bloc intensif" designation
69	other (please specify)

DROP INS	
70	forgotten on original test registration
71	late registration to course
72	transferred in from another school/province/country
73	re-write (not currently enrolled)
79	other (please specify)

MIGRANCY	
90	transferred to another school within the province
91	transferred to another school outside of the province
99	other (please specify)

Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Part 1

June 2004

Manitoba
Building for the Future

Senior 4 Pre-Calculus Mathematics (40S)
Standards Test

DESCRIPTION

Total Possible Marks: 100 **Time: 3 hours**

Part	Description	Time	Value
Part 1	• **Open Response:** 10 questions worth a total of 34 marks Calculators (scientific or graphing) are permitted.	60 minutes	34%
Part 2	• **Selected Response:** 15 questions worth 1 mark each • **Restricted Response:** 15 questions worth 1 mark each • **Open Response:** 13 questions worth a total of 36 marks Calculators are not permitted.	120 minutes	66%

GENERAL DIRECTIONS

- Formulas are given on the "Formula Sheet" at the beginning of this test booklet. Please do not tear out any pages other than the "Formula Sheet".
- Read all instructions carefully.
- The blank pages at the back of each booklet may be used for rough work. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- When *Part 2* booklet is distributed, you may keep Part 1 in case you wish to continue working on Part 1 questions (without a calculator).

Pre-Calculus Mathematics (40S) Formula Sheet

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

$$e \approx 2.71828$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n\log_a M$$

$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$(y-k) = a(x-h)^2$$

$$(x-h) = a(y-k)^2$$

$$t_n = t_1 r^{n-1}$$

$$S_n = \frac{t_1(1-r^n)}{1-r} = \frac{t_1(r^n-1)}{r-1}$$

$$S_n = \frac{t_1 - t_n r}{1-r}$$

$$S_\infty = \frac{t_1}{1-r}, \quad |r| < 1$$

Open-Response Questions

Instructions

- There are 10 questions for a total of 34 marks.
- Calculators (scientific or graphing) are allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions should include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized and clear.
- If any curve contains asymptotes, the asymptotes should be included in the graph.

DO NOT USE

Open-Response Questions

1. Solve for θ. Give the general solution in radian measure correct to 3 decimal places. 101

$$3\sin^2\theta - 10\sin\theta - 8 = 0$$

(4 marks)

2. Solve the following equation algebraically. State your answer correct to 3 decimal places. 102

$$5^{2x-1} = 7^{x+4}$$

(3 marks)

DO NOT USE

3. In June 1998, the black bear population in Manitoba was estimated at 1500 bears. 103
The population can be modelled by the equation $A = Pe^{rt}$ where r is the annual rate of increase and t represents the number of years.
Find the annual rate of increase in the bear population if it was estimated at 1740 bears in June 2001.
State your answer correct to 3 decimal places.

(3 marks)

4. How many numbers between 99 and 999 are divisible by 5 and have no repetition of digits? 104

(3 marks)

DO NOT USE

5. A committee of 7 people is to be randomly chosen from a class of 10 students. Joyce, Lynn and Lori are 3 of the 10 students.
What is the probability that Joyce, Lynn and Lori will be on the committee?
State your answer as an exact value or correct to 3 decimal places. 105

(3 marks)

6. Find the simplified expression for the 6^{th} term in the expansion of: 106

$$\left(x^3 - \frac{4}{x}\right)^8$$

(3 marks)

DO NOT USE

7. How many different arrangements are possible using all the letters in the word SUCCESS if all the S's must be together? 107

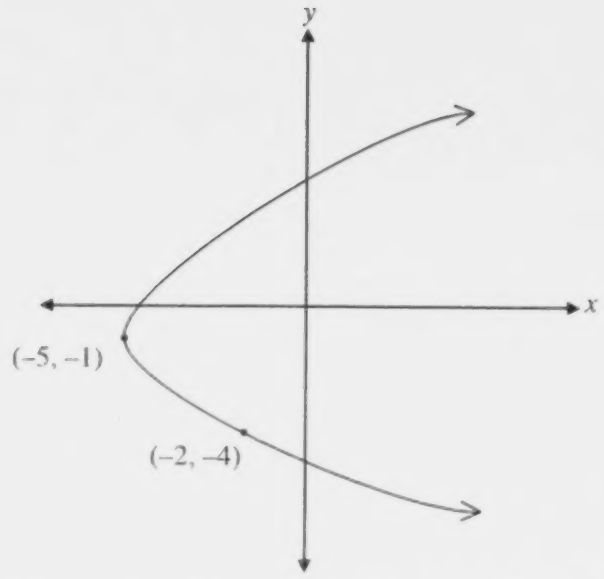
(3 marks)

8. Mr. and Mrs. Smith share the job of doing the laundry in their home. 108
Mr. Smith does the laundry 70% of the time and Mrs. Smith 30%.
The probability of clothes shrinking in the laundry is 15% when Mr. Smith does the laundry and 10% when Mrs. Smith does the laundry.
One day, the clothes from the laundry had shrunk.
Find the probability that Mr. Smith did the laundry that day.
State your answer correct to 3 decimal places.

(4 marks)

DO NOT USE

9.



a) Find the equation of the parabola with the vertex at (–5, –1) shown above. 109

(3 marks)

b) State the x-intercept as an exact value or correct to 3 decimal places. 110

(1 mark)

DO NOT USE

10. A conic is represented by the equation $x^2 + 9y^2 + 2x - 54y + 73 = 0$.

a) Find the centre of the conic. 111

(3 marks)

b) Give the domain of the conic. 112

(1 mark)

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

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Assessment and Evaluation Branch,
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Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Part 2

June 2004

Manitoba
Building for the Future

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Selected-Response Questions

Instructions

- There are 15 selected-response questions.
- Each question is worth 1 mark.
- Read each question carefully.
- Examine all the choices before you decide on your answer.
- You may use the spaces beside each question for rough work.
- Record your answers on the sheet provided.
- Provide only one answer per question.
- There is no penalty for guessing.
- Answer all questions.
- Calculators (scientific or graphing) are **not** allowed for this part of the test.

Selected-Response Questions

11. An angle of 20 radians is equivalent to an angle of:

a) $\frac{20^\circ}{\pi}$

b) $9\pi^\circ$

c) $\frac{\pi^\circ}{9}$

d) $\frac{3600^\circ}{\pi}$

12. The solutions to the equation $\csc\theta + 2 = 0$ in the interval $[0, 2\pi]$ are:

a) $\frac{\pi}{6}, \frac{5\pi}{6}$

b) $\frac{2\pi}{3}, \frac{4\pi}{3}$

c) $\frac{7\pi}{6}, \frac{11\pi}{6}$

d) no solution

13. If $\tan\theta = \frac{a}{b}$ and $\sin\theta = \frac{d}{c}$, then a possible expression for $\cos\theta$ is:

a) $\frac{bd}{ac}$

b) $\frac{ad}{bc}$

c) $\frac{ac}{bd}$

d) $1 - \left(\frac{d}{c}\right)^2$

14. The value of $\cos\frac{7\pi}{12}$ is equivalent to:

a) $\cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

b) $\cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

c) $\sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

d) $\sin\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{\pi}{4}$

15. The expression $\frac{\cot\theta}{\csc\theta}$ is equivalent to:

a) $\cos\theta$

b) $\sin\theta$

c) $\frac{\cos\theta}{\sin^2\theta}$

d) $\frac{\cos^2\theta}{\sin\theta}$

16. The point (–3, 8) is on the graph of $y = f(x)$.
On the graph $y = 2f(3x)$, this point will be transformed to:

a) (–9, 4)

b) (–1, 16)

c) (–6, 24)

d) (–9, 16)

17. A cosine function has a period of 3 and a range of $[-4, 6]$.

A possible equation for this function is:

a) $y = 5\cos\left(\frac{\pi}{3}x\right) - 1$

b) $y = 5\cos\left(\frac{\pi}{3}x\right) + 1$

c) $y = 5\cos\left(\frac{2\pi}{3}x\right) - 1$

d) $y = 5\cos\left(\frac{2\pi}{3}x\right) + 1$

18. The expression $\log_5\left[\frac{\sqrt{x}}{x^4(x+2)}\right]$ is equivalent to:

a) $\frac{\log_5 x^{\frac{1}{2}}}{\log_5 x^4 + \log_5(x+2)}$

b) $\frac{1}{2}\log_5 x - \log_5(x^4 + x + 2)$

c) $\frac{1}{2}\log_5 x - 4\log_5 x + \log_5(x+2)$

d) $\frac{1}{2}\log_5 x - 4\log_5 x - \log_5(x+2)$

19. When Joe was born, his father invested $800 on the condition that it would not be withdrawn until Joe is 18 years old. Assuming that the interest rate remains at 12%, compounded quarterly, which of the following expressions would represent the value after 18 years?

a) $800(1.04)^{54}$

b) $800(1.03)^{18}$

c) $800(1.03)^{72}$

d) $800(1.12)^{18}$

20. The value of $\log_2 \frac{1}{32}$ is:

a) -5

b) $-\frac{1}{5}$

c) $\frac{1}{5}$

d) 5

21. A class consists of 12 girls and 15 boys. In how many ways can we select a group of 5 girls and 4 boys from this class?

a) ${}_{27}C_9$

b) ${}_{12}C_5 + {}_{15}C_4$

c) ${}_{12}C_5 \cdot {}_{15}C_4$

d) ${}_{27}C_5 + {}_{27}C_4$

22. In how many ways can 5 beads of different colors be arranged on a bracelet?

a) $4!$

b) $\frac{4!}{2}$

c) $5!$

d) $\frac{5!}{2}$

23. The probability that an event will occur is $\frac{3}{8}$. What is the probability that the event will **not** occur on the first trial and **not** occur on the second trial?

a) $\frac{25}{64}$

b) $\frac{5}{8}$

c) $\frac{55}{64}$

d) $\frac{9}{64}$

24. A bag contains 3 red and 3 blue marbles. If 3 marbles are chosen without replacement, what is the probability that they will be all red or all blue?

a) $\frac{1}{20}$

b) $\frac{1}{10}$

c) $\frac{1}{8}$

d) $\frac{1}{2}$

25. The range of the relation $\frac{x^2}{16} - \frac{y^2}{25} = 1$ is:

a) $(-\infty, \infty)$

b) $[-5, 5]$

c) $(-\infty, -5] \cup [5, \infty)$

d) $(-\infty, -4] \cup [4, \infty)$

Restricted-Response Questions

Instructions

- There are 15 restricted-response questions.
- Each question is worth 1 mark.
- Calculators are **not** allowed for this part of the test.
- Write your final answer in the space provided.
- Work need not be shown in order to obtain full marks for a correct answer. However, in the case of an incorrect final answer, part marks may be awarded only if work is shown.
- Solutions need not have rationalized denominators.

DO NOT USE

Restricted-Response Questions

26. Express $\dfrac{\cos\theta \tan\theta + \sin\theta}{\cos\theta}$ as a single trigonometric function of θ. 113

(1 mark)

27. On a circle, the length of the arc intercepted by a central angle θ is 60 cm. If the radius of the circle is 20 cm, state the measure of θ in radians. 114

(1 mark)

DO NOT USE

28. Give a value of x in the interval $[0, 2\pi]$ for which the function $f(x) = 2\sec x$ is undefined. 115

(1 mark)

29. For the function $f(x) = 5\cos[\pi(x-3)] + 2$, find the value of $f(12)$. 116

(1 mark)

DO NOT USE

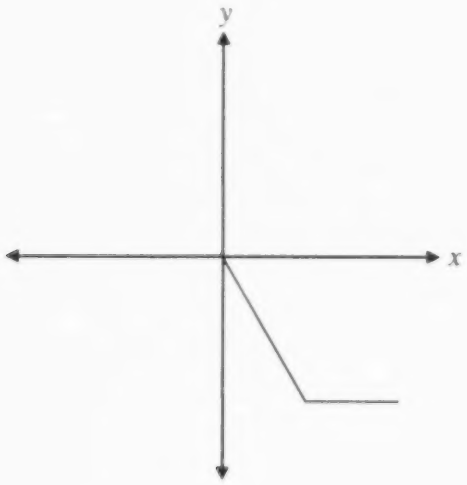
30. The x-intercept of $y = f(x)$ is 5. 117

State the x-intercept of $y = f\left(\frac{x}{3}\right)$.

(1 mark)

31. Complete the graph of $f(x)$ below if $f(x) = f(-x)$. 118

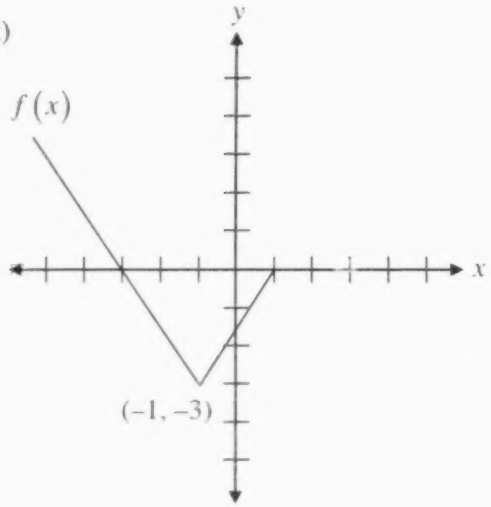
(1 mark)

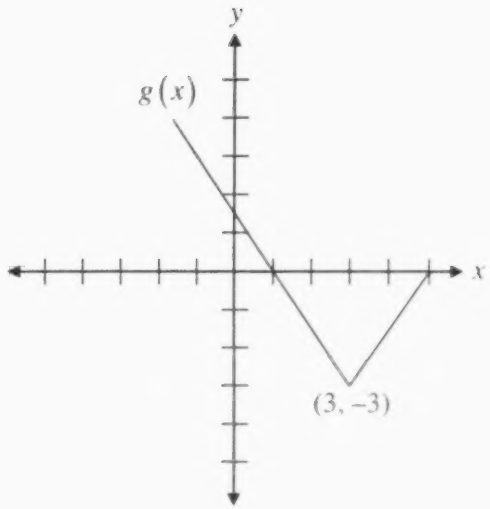


DO NOT USE

32. Express $g(x)$ in terms of f.

(1 mark)





$g(x) =$ _____

119

33. If $\log_a x = 16$, find the value of $\log_a \sqrt{x}$.

120

(1 mark)

DO NOT USE

34. Find $f^{-1}(x)$ if $f(x) = \log x$.

(1 mark)

$f^{-1}(x) =$ _____ 121

35. The first term of a geometric sequence is x^{10} and the sixth term is x^{25}. 122
Find the common ratio.

(1 mark)

DO NOT USE

36. In how many ways can 8 people be seated at a round table with 8 chairs if three of the people insist on being seated together? Leave your answer in factorial form. 123

(1 mark)

37. In the binomial expansion of $(a+b)^{15}$, what is the numerical coefficient of the term containing a^2b^{13}? 124

(1 mark)

DO NOT USE

38. The probability that Samir brings a pencil to class is 0.5. The probability that he brings a calculator to class is 0.4. What is the probability that he brings a pencil and a calculator to class? 125

(1 mark)

39. In how many ways can 3 people be seated in a row of 5 chairs?
State your answer as a whole number. 126

(1 mark)

DO NOT USE

127

40. Identify the conic whose equation is $x^2 - y^2 - 5x + 1 = 0$.

(1 mark)

Open-Response Questions

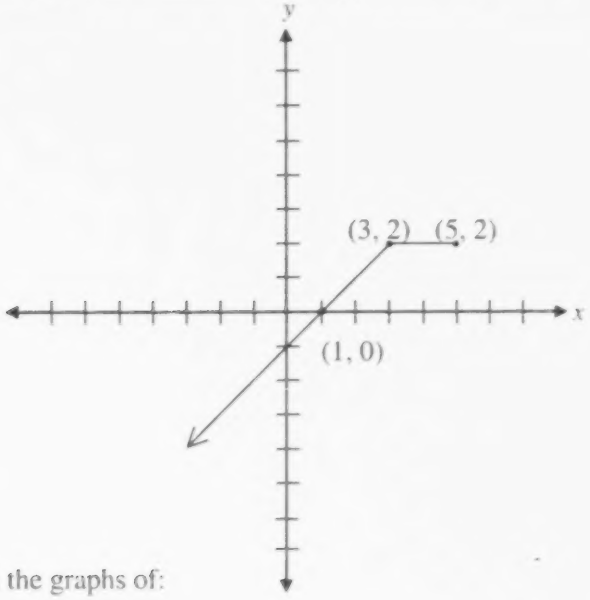
Instructions

- There are 13 questions worth a total of 36 marks.
- Calculators are **not** allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear and well organized.
- If any curve contains asymptotes, the asymptotes should be included in the graph.
- Solutions need not have rationalized denominators.

DO NOT USE

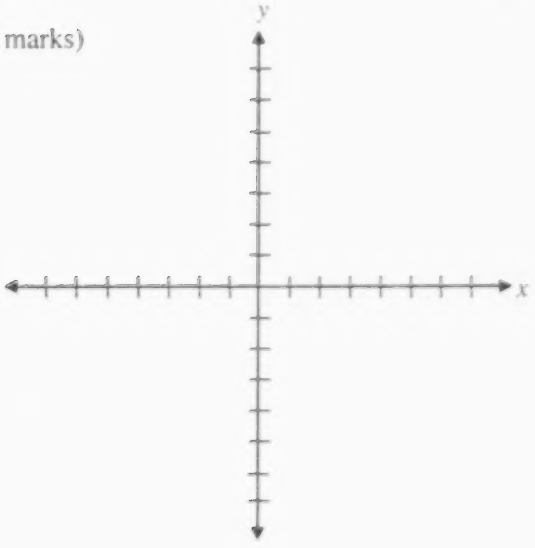
Open-Response Questions

41. The graph of a function $y = f(x)$ is shown below.

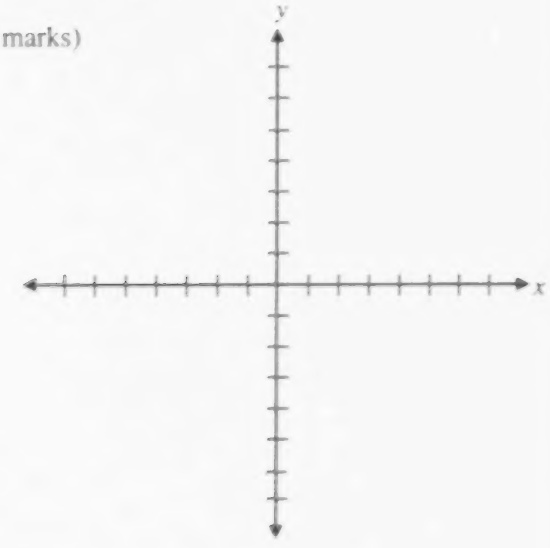


Sketch the graphs of:

a) $y = -f(x+1)$ (2 marks)



b) $y = \dfrac{1}{f(x)}$ (2 marks)



128

129

DO NOT USE

42. Solve for θ in the interval $[\pi, 2\pi]$: 130

$$(\tan\theta + 1)(2\cos\theta - 1) = 0$$

(2 marks)

43. Find the exact value of: 131

a) $\sin\frac{11\pi}{12}$

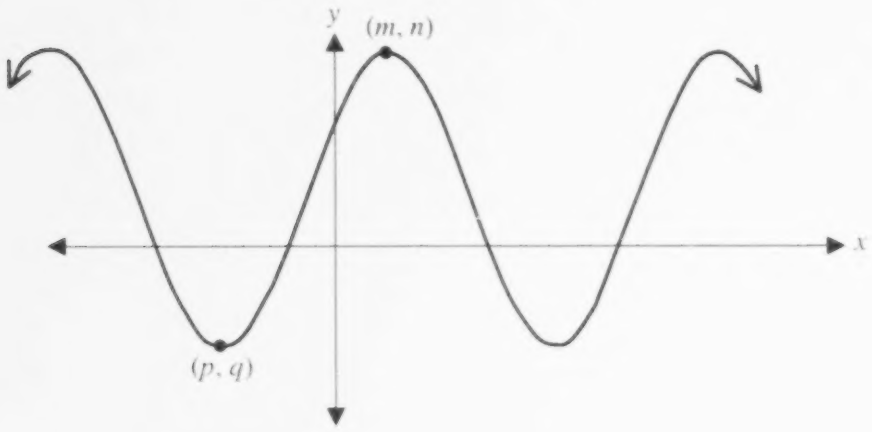
(3 marks)

b) $\csc\frac{11\pi}{12}$ 132

(1 mark)

DO NOT USE

44. The graph of $y = 4\cos\left(x - \frac{\pi}{4}\right) + 2$ is sketched below.



Find the exact values of m, n and p.

(3 marks)

$m =$ _____

$n =$ _____

$p =$ _____

133

45. Prove the following identity:

$$\frac{\tan\theta \cdot \csc^2\theta}{\tan^2\theta + 1} = \cot\theta$$

(3 marks)

DO NOT USE

46. If $\tan\theta = -\frac{4}{3}$ and $\frac{\pi}{2} < \theta < \pi$, state the exact value of $\cos 2\theta$. 135

(3 marks)

47. Show that $f(x) = x^3 - x$ is an odd function. 136

(2 marks)

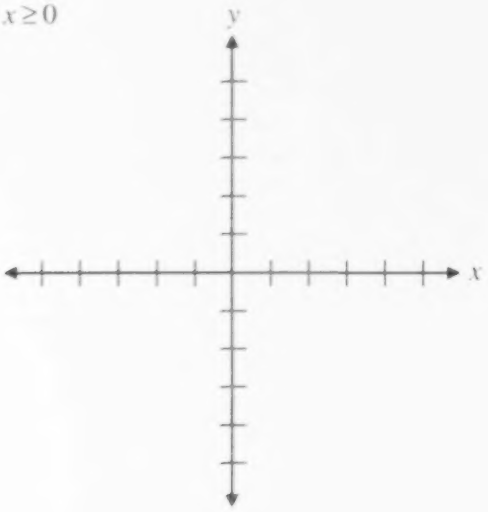
DO NOT USE

137

48. Sketch the graph of $f(x)$ on the axes below.

$$f(x)=\begin{cases}-x, & x<0\\ \sqrt{x}, & x\geq 0\end{cases}$$

(2 marks)



138

49. Solve for x. Give your answer in simplest form.

$$\log_9(\log_4 x)=\frac{1}{2}$$

(2 marks)

DO NOT USE

50. Evaluate $\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{k}$. 139

(2 marks)

51. Find the x-intercept(s) of the following function: 140

$$y = \log(10 - 3x) - 2\log x$$

(4 marks)

DO NOT USE

52. Solve for n:

$$ {}_nP_2 = 12 $$

(2 marks)

141

53. Solve for x over $[0, 2\pi]$:

$$ \cos 2\theta = \frac{\sqrt{3}}{2} $$

(3 marks)

142

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

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NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Marking Guide

June 2004

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Assessment and Evaluation Branch,
Winnipeg, Manitoba R3E 3J5

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General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by department staff prior to sample marking should the booklet be selected.

Please ensure that:

- the student test booklet number and the number on the *Answer/Scoring Sheet* are identical;
- **only a pencil is used to complete the *Answer/Scoring Sheets*, by both students and markers;**
- the totals of each of the four parts are written at the bottom;
- each student's final result is recorded, by test booklet number, on the corresponding ***Answer/Scoring Sheet***.
- the grand total is written in the appropriate location;
- the *Answer/Scoring Sheet* is complete; and
- a photocopy has been made for school records.

Once marking is completed, please forward the ***Answer/Scoring Sheets*** to Manitoba Education, Citizenship and Youth in the envelope provided (for more information see the *Administration Manual for Supervising Teachers*).

Scoring the Selected-Response Questions

Please follow the instructions on the ***Answer Key for Selected-Response Questions*** on page 5.

Scoring the Restricted-Response and Open-Response Questions

The remainder of Senior 4 Pre-Calculus Mathematics (40S) Standards Test is composed of restricted-response questions and open-response questions. Restricted-response questions are worth 1 mark each and open-response questions are worth 2 to 5 marks each.

These questions are designed to elicit well-defined responses according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with each question.

To receive full marks, a student's response must be complete and correct. Otherwise, 1 mark must be deducted for each concept error, and ½ mark must be deducted for a calculation or arithmetic error. Where alternative answering methods are possible, the *Marking Key* attempts to address the most common solutions.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education, Citizenship and Youth at the earliest opportunity to advise and/or seek assistance. If required, we will connect you to an advisor. Also, please be sure to notify us before making any modifications to the answer keys or scoring rubrics. Contact person: Chantal Bérard at 945-7590 or toll-free at 1 800 282-8069, extension 7590.

SCORING GUIDELINES

Answer Key for Selected-Response Questions

Answer Key for Selected-Response Questions

If you are using this sheet by punching out holes corresponding to the correct answers and overlaying this sheet on the *Answer/Scoring Sheet*, then

1) check the student's responses for multiple bubbles first (questions with multiple bubbles are to be scored as "0");

2) overlay this page on the *Answer/Scoring Sheet*; and

3) count the number of correct responses, excluding questions with multiple responses, if any.

You may also score the selected-response questions by making a transparency of this page and overlaying it on the *Answer/Scoring Sheet*.

Remember to write the total score for the selected-response questions at the bottom of the *Answer/Scoring Sheet*.

SELECTED RESPONSE QUESTIONS / QUESTIONS À RÉPONSE CHOISIE

Fill in the best answer for each question. / Choisir la meilleure réponse pour chaque question.

11	Ⓐ Ⓑ Ⓒ **Ⓓ**	16	Ⓐ **Ⓑ** Ⓒ Ⓓ	21	Ⓐ Ⓑ **Ⓒ** Ⓓ
12	Ⓐ Ⓑ **Ⓒ** Ⓓ	17	Ⓐ Ⓑ Ⓒ **Ⓓ**	22	Ⓐ **Ⓑ** Ⓒ Ⓓ
13	**Ⓐ** Ⓑ Ⓒ Ⓓ	18	Ⓐ Ⓑ Ⓒ **Ⓓ**	23	**Ⓐ** Ⓑ Ⓒ Ⓓ
14	Ⓐ **Ⓑ** Ⓒ Ⓓ	19	Ⓐ Ⓑ **Ⓒ** Ⓓ	24	Ⓐ **Ⓑ** Ⓒ Ⓓ
15	**Ⓐ** Ⓑ Ⓒ Ⓓ	20	**Ⓐ** Ⓑ Ⓒ Ⓓ	25	**Ⓐ** Ⓑ Ⓒ Ⓓ

Selected-Response Questions			
Question	*Outcome*	*Question*	*Outcome*
11	A1	19	D8
12	A4	20	D4
13	C1	21	E3
14	C2	22	E1
15	C1	23	G3
16	B2	24	G2
17	B7	25	F3
18	D5		

SCORING GUIDELINES

Marking Key
including student exemplars

Part 1—Open-Response Questions

Question No. 1

Outcome: A5

Question

1. Solve for θ. Give the general solution in radian measure correct to 3 decimal places. **4 marks**

$$3\sin^2\theta - 10\sin\theta - 8 = 0$$

Solution

Method 1

$(3\sin\theta + 2)(\sin\theta - 4) = 0$

$\sin\theta = -\frac{2}{3}$ $\quad \sin\theta = 4$ — ½ mark for solving for $\sin\theta$

$\downarrow$

no solution — ½ mark for eliminating extraneous root

$\sin^{-1}\left(-\frac{2}{3}\right) = -0.730$ — ½ mark

Reference arc $= 0.730$ — ½ mark for θ_R

$\theta = 3.871 + 2k\pi$, where $k \in I$

$\theta = 5.553 + 2k\pi$, where $k \in I$

} 2 marks: 1 for θ; 1 for general solution

Method 2 (graphing calculator)

$Y_1 = 3\sin^2\theta - 10\sin\theta - 8$

Find a zero to use for θ_R

} 1 mark for explanation of process

Reference arc $= 0.730$ — 1 mark for θ_R

$\theta = 3.871 + 2k\pi$, where $k \in I$

$\theta = 5.553 + 2k\pi$, where $k \in I$

} 2 marks: 1 for θ; 1 for general solution

Note(s)

- give 3 marks for a correct solution in degrees (with all degree symbols)
  i.e., $221.810° + 360°k,\ k \in I$

  $318.190° + 360°k,\ k \in I$
- **deduct** ½ mark for not stating $k \in I$
- give full marks if correct solution is given without stating reference arc

EXEMPLARS

1. Solve for θ. Give the general solution in radian measure correct to 3 decimal places.

$$3\sin^2\theta - 10\sin\theta - 8 = 0$$

$(3\sin\theta + 2)(\sin\theta - 4) = 0$

$\sin\theta = -2/3, \cancel{4}$ $[\sin^{-1}(-2/3]$ in radien mode

$\theta = -0.730$

Mark:	**1½ out of 4**
Note(s):	• gave ½ mark for solving for $\sin\theta$ • gave ½ mark for eliminating extraneous root • gave ½ mark for $\theta = -0.730$

Question No. 2

Outcome: D6

Question

2. Solve the following equation algebraically. State your answer correct to 3 decimal places.

$5^{2x-1} = 7^{x+4}$ 3 marks

Solution

$(2x-1)\log 5 = (x+4)\log 7$ 1 mark for log theorem

$2x\log 5 - \log 5 = x\log 7 + 4\log 7$ 1 mark for distribution

$2x\log 5 - x\log 7 = 4\log 7 + \log 5$

$x(2\log 5 - \log 7) = 4\log 7 + \log 5$

$x = \dfrac{4\log 7 + \log 5}{2\log 5 - \log 7}$ ½ mark for isolating x

$x = 7.379$ ½ mark for answer

Note(s)

- $\log 7 = 0.845098$ $\log 5 = 0.69897$ $\ln 7 = 1.94591$ $\ln 5 = 1.60944$
- **deduct** ½ mark if parentheses are assumed but not written in step 1
- **deduct** ½ mark for incorrect decimal places

EXEMPLARS

2. Solve the following equation algebraically. State your answer correct to 3 decimal places.

$5^{2x-1} = 7^{x+4}$

$\log 5^{2x-1} = \log 7^{x+4}$

$(2x-1)\log 5 = (x+4)\log 7$

$\log 10x - \log 5 = \log 7x + \log 28$

$\log 10x - \log 7x = \log 28 + \log 5$

$\frac{x(\log 10 - \log 7)}{(\log 10 - \log 7)} = \frac{\log 28 + \log 5}{(\log 10 - \log 7)}$

$x = \frac{2.146128}{0.154902}$

$\boxed{x = 13.855}$

Mark: **1½ out of 3**

Note(s):
- gave 1 mark for log theorem
- gave ½ mark for consistent answer

Question No. 3

Outcome: D7

Question

3. In June 1998, the black bear population in Manitoba was estimated at 1500 bears.
The population can be modelled by the equation $A = Pe^{rt}$ where r is the annual rate of increase and t represents the number of years.
Find the annual rate of increase in the bear population if it was estimated at 1740 bears in June 2001.
State your answer correct to 3 decimal places. **3 marks**

Solution

$1740 = 1500e^{3r}$ — 1 mark for correct substitution

$\frac{1740}{1500} = e^{3r}$

$\ln\left(\frac{1740}{1500}\right) = 3r \ln e$ — 1 mark for log theorem

$r = \frac{\ln\left(\frac{1740}{1500}\right)}{3}$ — ½ mark for isolating r

$= 0.049$ — ½ mark for answer

Note(s)

- **deduct** ½ mark for incorrect decimal places

EXEMPLARS

3. In June 1998, the black bear population in Manitoba was estimated at 1500 bears.
The population can be modelled by the equation $A = Pe^{rt}$ where r is the annual rate of increase and t represents the number of years.
Find the annual rate of increase in the bear population if it was estimated at 1740 bears in June 2001.
State your answer correct to 3 decimal places.

$A = Pe^{rt}$

$\frac{1740}{1500} = \frac{1500e^{r(4)}}{1500}$

$1.16 = e^{r4}$

$\ln 1.16 = r4 \ln e$ ($\ln e = 1$)

$\frac{\ln 1.16}{4} = \frac{r4}{4}$

$\frac{\ln 1.16}{4} = r$

$r = 0.037$

Mark: **2½ out of 3**
Note(s): • incorrect number of years

Question No. 4

Outcome: E1

Question

4. How many numbers between 99 and 999 are divisible by 5 and have no repetition of digits?

3 marks

Solution

Case 1 $\underline{9} \times \underline{8} \times \underline{1}_{\{0\}} = 72$ 1 mark for Case 1

Case 2 $\underline{8} \times \underline{8} \times \underline{1}_{\{5\}} = 64$ 1 mark for Case 2

TOTAL = $72 + 64$

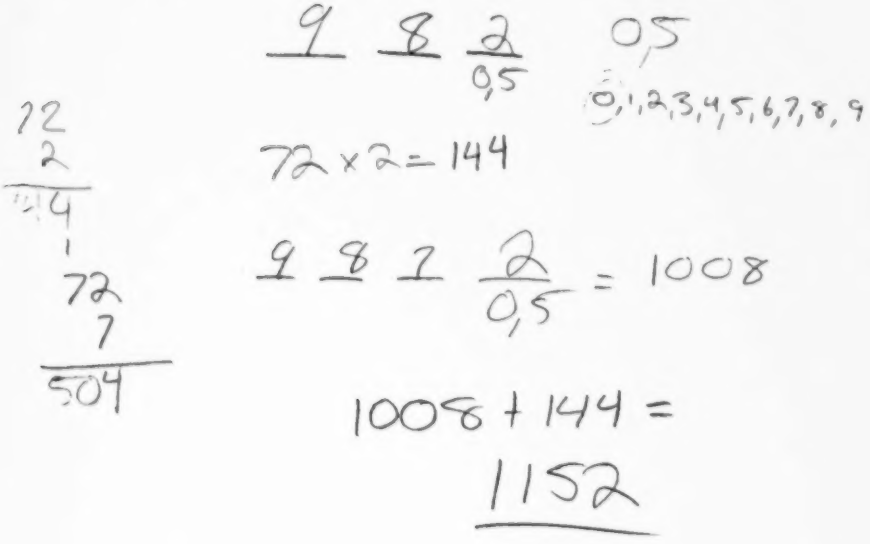
$= 136$ 1 mark for addition of cases

Note(s)

- **deduct** ½ mark if description of cases is not given

EXEMPLARS

4. How many numbers between 99 and 999 are divisible by 5 and have no repetition of digits?



Mark:	**1 out of 3**
Note(s):	• gave 1 mark for addition of cases

Question No. 5

Outcome: G5

Question

5. A committee of 7 people is to be randomly chosen from a class of 10 students.
Joyce, Lynn and Lori are 3 of the 10 students.
What is the probability that Joyce, Lynn and Lori will be on the committee?
State your answer as an exact value or correct to 3 decimal places.

3 marks

Solution

There are ${}_{10}C_7$ total possible committees.

If Joyce, Lynn and Lori are on the committee, there are ${}_7C_4$ possible committees.

$\frac{{}_7C_4}{{}_{10}C_7}$ 2 marks (1 mark for ${}_7C_4$; 1 mark for ${}_{10}C_7$)

$= \frac{35}{120}$

$= \frac{7}{24}$ OR $0.29\overline{16}$ OR 0.292 1 mark for answer

EXEMPLARS

5. A committee of 7 people is to be randomly chosen from a class of 10 students.
Joyce, Lynn and Lori are 3 of the 10 students.
What is the probability that Joyce, Lynn and Lori will be on the committee?
State your answer as an exact value or correct to 3 decimal places.

Sample space
$_{10}C_7 \cdot 120$
$\frac{3^1}{10_5} \cdot \frac{2^1}{9_3} \cdot \frac{1}{8} = \boxed{\frac{1}{120}}$ is the probability
that they will all be on the committee.

Mark: **1 out of 3**
Note(s): • gave 1 mark for $_{10}C_7$

Question No. 6

Outcome: E4

Question

6. Find the simplified expression for the 6^{th} term in the expansion of:

3 marks

$$\left(x^3 - \frac{4}{x}\right)^8$$

Solution

$${}_8C_5\left(x^3\right)^3\left(\frac{-4}{x}\right)^5$$

1½ mark for setup

- ½ mark for ${}_8C_5$
- ½ mark for $\left(x^3\right)^3$
- ½ mark for $\left(-\frac{4}{x}\right)^5$

$$56x^9 \bullet \frac{-1024}{x^5}$$

1 mark (½ mark for $56x^9$; ½ mark for $-\frac{1024}{x^5}$)

$$= -57\,344x^4$$

½ mark for answer

Note(s)

- second step is optional
- if student uses ${}_8C_6$ give maximum 2 marks

$${}_8C_6\left(x^3\right)^2\left(-\frac{4}{x}\right)^6 = 114\,688$$

- ${}_8C_3$ can be written instead of ${}_8C_5$

EXEMPLARS

6. Find the simplified expression for the 6th term in the expansion of:

$$\left(x^3 - \frac{4}{x}\right)^8$$

$$nC_{r-1}\, x^{n-r+1}\, y^{r-1}$$

$$8C_{6-1}\,(x^3)^{8-6+1}\,(4/x)^{6-1}$$

$$8C_5\,(x^3)^3\,(4/x)^5$$

$$= 56\,x^6\,\frac{4}{x}^5$$

Mark:	**1 out of 3**
Note(s):	• gave ½ mark for ${}_8C_5$
	• gave ½ mark for $\left(x^3\right)^3$

Question No. 7

Outcome: G2

Question

7. How many different arrangements are possible using all the letters in the word SUCCESS if all the S's must be together?

3 marks

Solution

Place S's together : (S S S) U C C E

No. of arrangements $= \frac{5!}{2!}$ 1 mark for 5!, 1 mark for 2!

$= \frac{120}{2}$

$= 60$ 1 mark for answer

EXEMPLARS

7. How many different arrangements are possible using all the letters in the word SUCCESS if all the S's must be together?

$$\frac{7!}{3!\,2!}$$

$$= 420$$

Mark: **1 out of 3**
Note(s): • gave 1 mark for consistent answer

Question No. 8

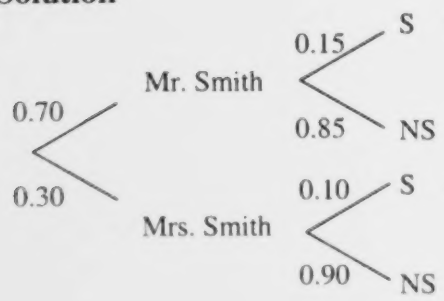
Outcome: G4

Question

8. Mr. and Mrs. Smith share the job of doing the laundry in their home.
Mr. Smith does the laundry 70% of the time and Mrs. Smith 30%.
The probability of clothes shrinking in the laundry is 15% when Mr. Smith does the laundry and 10% when Mrs. Smith does the laundry.
One day, the clothes from the laundry had shrunk.
Find the probability that Mr. Smith did the laundry that day.
State your answer correct to 3 decimal places.

4 marks

Solution



$= (0.70)(0.15) = 0.105$

1 mark
for probability P (Mr. S and shrink)

$= (0.30)(0.10) = 0.03$

1 mark
for probability P (Mrs. S and shrink)

$$P(\text{Mr. Smith}) = \frac{0.105}{0.105 + 0.03}$$

1 mark for probability fraction

$$= \frac{0.105}{0.135}$$

$$= 0.778$$

1 mark for answer

Method 2

$$\frac{0.7 \times 0.15}{0.7 \times 0.15 + 0.3 \times 0.1}$$

3 marks (1 mark for each of the two correct product and 1 mark for the correct setup)

$$= \frac{0.105}{0.135}$$

$$= 0.778$$

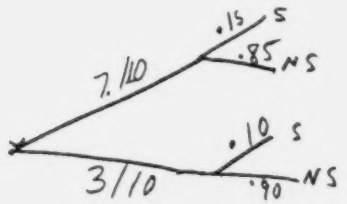
1 mark for answer

Note(s)

- if only a correct tree diagram is given, give 1 mark
- **deduct** ½ mark for incorrect decimal places

EXEMPLARS

8. Mr. and Mrs. Smith share the job of doing the laundry in their home.
Mr. Smith does the laundry 70% of the time and Mrs. Smith 30%.
The probability of clothes shrinking in the laundry is 15% when Mr. Smith does the laundry and 10% when Mrs. Smith does the laundry.
One day, the clothes from the laundry had shrunk.
Find the probability that Mr. Smith did the laundry that day.
State your answer correct to 3 decimal places.



$= (.7 \times .15) + (.3 \times .10)$

$= .105 + .03$

$= 0.108$ = the probability that Mr. Smith did the laundry.

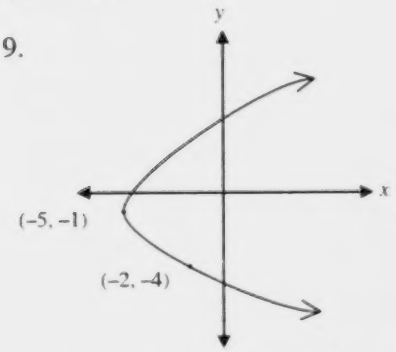
Mark:	**2 out of 4**
Note(s):	• gave 1 mark for probability of (Mr. S and shrink) • gave 1 mark for probability of (Mrs. S and shrink)

Question No. 9

Outcome: a) F2 b) F3

Question

9.



a) Find the equation of the parabola with the vertex at (–5, –1) shown above. — 3 marks

b) State the x-intercept as an exact value or correct to 3 decimal places. — 1 mark

Solution

a) $x-h=a(y-k)^2$

$(x+5)=a(y+1)^2$ — 1 mark for substituting coordinates of vertex

$(-2+5)=a(-4+1)^2$ — 1 mark for substituting coordinates of (–2, –4)

$(3)=a(-3)^2$

$3=9a$

$\frac{1}{3}=a$ — ½ mark for solving for a

$(x+5)=\frac{1}{3}(y+1)^2$

OR

$3x-y^2-2y+14=0$

} ½ mark for answer

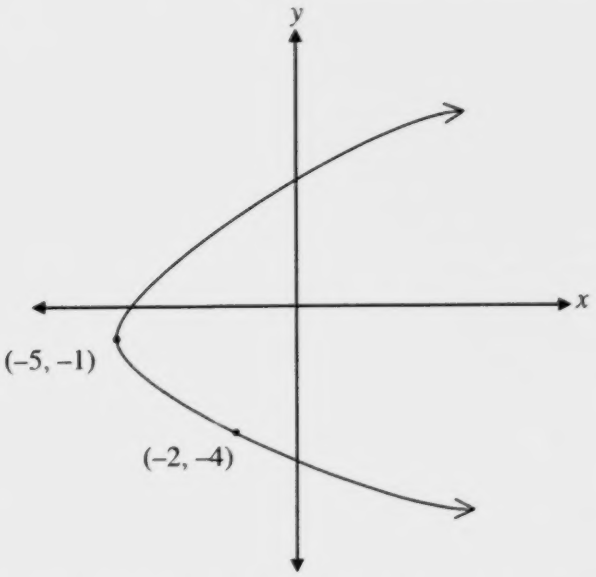
b) x-intercept

$3x+14=0$ — ½ mark for substituting $y = 0$

$x=-\frac{14}{3}$ — ½ mark for solving x consistent with answer to part a

EXEMPLARS

9.



a) Find the equation of the parabola with the vertex at (–5, –1) shown above.

$y = a(x-h)^2 + k$

$-4 = a(-2-(-5))^2 + (-1)$

$-4 = a(9) + -1$

$\frac{-3}{9} = \frac{9a}{9}$

$a = -\frac{1}{3}$ ∴

Eqn: $y = -\frac{1}{3}(x+5)^2 - 1$

Mark:	**2 out of 3**
Note(s):	• deducted 1 mark for incorrect equation (conceptual error)

b) State the x-intercept as an exact value or correct to 3 decimal places.

→ There is no x-intercept?
- graph point down and the vertex is one down from the x axis

Mark:	**0 out of 1**
Note(s):	• answer from equation and not graph

Question No. 10

Outcome: F2

Question

10. A conic is represented by the equation $x^2 + 9y^2 + 2x - 54y + 73 = 0$.

    a) Find the centre of the conic. — 3 marks

    b) Give the domain of the conic. — 1 mark

Solution

a)

$$x^2 + 2x + 9y^2 - 54y = -73$$
$$x^2 + 2x + 1 + 9\left(y^2 - 6y + 9\right) = -73 + 1 + 81$$
$$(x+1)^2 + 9(y-3)^2 = 9$$
$$\frac{(x+1)^2}{9} + (y-3)^2 = 1$$

2 marks
- 1 mark for completing the square for y (½ mark for left side and ½ mark for right side)
- 1 mark for completing the square for x (½ mark for left side and ½ mark for right side)

centre: $(-1, 3)$ — 1 mark for centre (½ mark for each coordinate)

b) $-4 \le x \le 2$ OR $[-4, 2]$ — 1 mark for interval

Note(s)

- b) ½ mark for $(-4, 2]$ or $[-4, 2)$ or $(-4, 2)$ or
  $-4 < x \le 2$ or $-4 \le x < 2$ or $-4 < x < 2$
- mark b) consistent with a)

EXEMPLARS

10. A conic is represented by the equation $x^2 + 9y^2 + 2x - 54y + 73 = 0$.

a) Find the centre of the conic.

$(x^2 + 2x + 1) + 9(y^2 - 6y + 9) = -73 + 1 + 81$

$\frac{(x+1)^2}{81} + \frac{9(y-3)^2}{81} = 1$

$\frac{(x+1)^2}{81} + \frac{(y-3)^2}{9} = 1$

$C = (-1, 3)$

Mark:	**2½ out of 3**
Note(s):	• deducted ½ mark for arithmetic error

b) Give the domain of the conic.

$\{x \mid -11 \leq x \leq 7; x \in \mathbb{R}\}$

Mark:	**0 out of 1**
Note(s):	• incorrect domain

Part 2—Restricted-Response Questions

Question No. 26

Outcome: C1

Question

26. Express $\dfrac{\cos\theta \tan\theta + \sin\theta}{\cos\theta}$ as a single trigonometric function of θ. 1 mark

Answer

$2\tan\theta$

Note(s)

- give ½ mark if change $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Question No. 27

Outcome: A1

Question

27. On a circle, the length of the arc intercepted by a central angle θ is 60 cm. If the radius of the circle is 20 cm, state the measure of θ in radians.

1 mark

Answer

$\theta = \frac{s}{r} = \frac{60 \text{ cm}}{20 \text{ cm}} = 3$ radians

Note(s)

- give ½ mark for $60 = 20(\theta)$

Question No. 28

Outcome: A6

Question

28. Give a value of x in the interval $[0, 2\pi]$ for which the function $f(x) = 2\sec x$ is undefined.

1 mark

Answer

$\frac{\pi}{2}$ OR $\frac{3\pi}{2}$

Note(s)

- give ½ mark if student states that the function is undefined when $\cos x = 0$
- give ½ mark if student solves $f(x) = \frac{2}{\sin x}$ which is undefined where $\sin x = 0$, $x = 0$, π or 2π

Question No. 29

Outcome: B7

Question

29. For the function $f(x) = 5\cos\left[\pi(x-3)\right]+2$, find the value of $f(12)$. 1 mark

Answer

$$\begin{aligned} f(12) &= 5\cos\left[\pi(12-3)\right]+2 \\ &= 5\cos(9\pi)+2 \\ &= 5(-1)+2 \\ &= -3 \end{aligned}$$

Question No. 30

Outcome: B2

Question

30. The x-intercept of $y = f(x)$ is 5.

State the x-intercept of $y = f\left(\frac{x}{3}\right)$.

1 mark

Answer

15

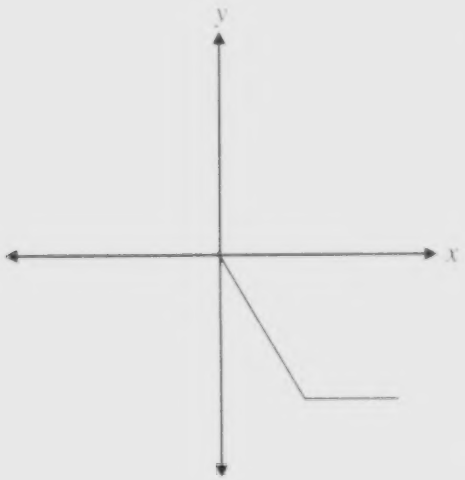
Question No. 31

Outcome: B3

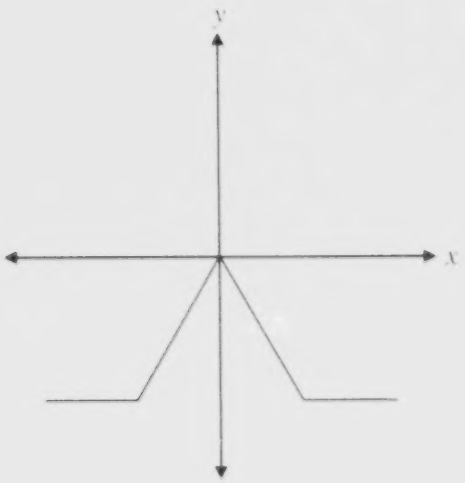
Question

31. Complete the graph of $f(x)$ below if $f(x) = f(-x)$.

1 mark



Answer



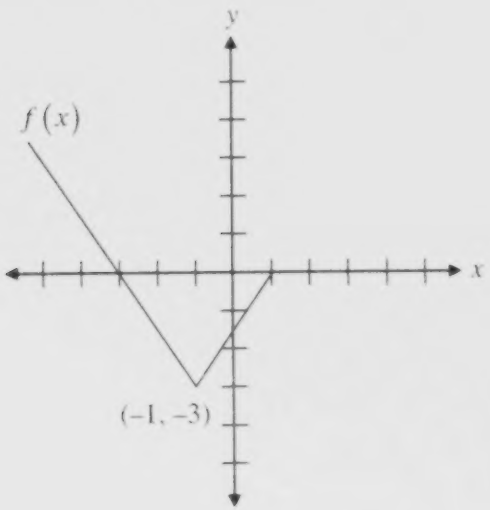
Question No. 32

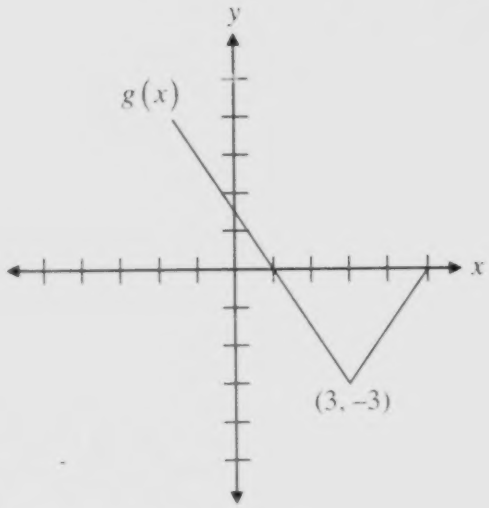
Outcome: B1

Question

32. Express $g(x)$ in terms of f.

1 mark





Answer

$g(x) = f(x-4)$

Question No. 33

Outcome: D5

Question

33. If $\log_a x = 16$, find the value of $\log_a \sqrt{x}$.

1 mark

Answer

$$\log_a \sqrt{x}$$
$$= \frac{1}{2}\log_a x$$
$$= \frac{1}{2}(16)$$
$$= 8$$

Note(s)

- give ½ mark for $\log_a \sqrt{x} = \frac{1}{2}\log_a x$

Question No. 34

Outcome: D7

Question

34. Find $f^{-1}(x)$ if $f(x) = \log x$.

1 mark

Answer

$f^{-1}(x) = 10^x$

Note(s)

- give ½ mark if $x = \log y$ stated

Question No. 35

Outcome: H1

Question

35. The first term of a geometric sequence is x^{10} and the sixth term is x^{25}. Find the common ratio.

1 mark

Answer

$x^{25} = x^{10} r^5$

$r^5 = x^{15}$

$r = x^3$

Note(s)

- give ½ mark for $r^5 = x^{15}$

Question No. 36

Outcome: E1

Question

36. In how many ways can 8 people be seated at a round table with 8 chairs if three of the people insist on being seated together? Leave your answer in factorial form.

1 mark

Answer

5!3!

Note(s)

- give ½ mark for 7!3! or 6!3! or 5!3 or 5!

Question No. 37

Outcome: E4

Question

37. In the binomial expansion of $(a+b)^{15}$, what is the numerical coefficient of the term containing a^2b^{13}?

1 mark

Answer

${}_{15}C_{13}$ OR 105 OR ${}_{15}C_{2}$

Question No. 38

Outcome: G2

Question

38. The probability that Samir brings a pencil to class is 0.5. The probability that he brings a calculator to class is 0.4. What is the probability that he brings a pencil and a calculator to class?

1 mark

Answer

$(0.5)(0.4) = 0.20$

Question No. 39

Outcome: E1

Question

39. In how many ways can 3 people be seated in a row of 5 chairs?
State your answer as a whole number.

1 mark

Answer

$\underline{5}\ \underline{4}\ \underline{3} = 60$ ways

Question No. 40

Outcome: F3

Question

40. Identify the conic whose equation is $x^2 - y^2 - 5x + 1 = 0$. 1 mark

Answer

hyperbola

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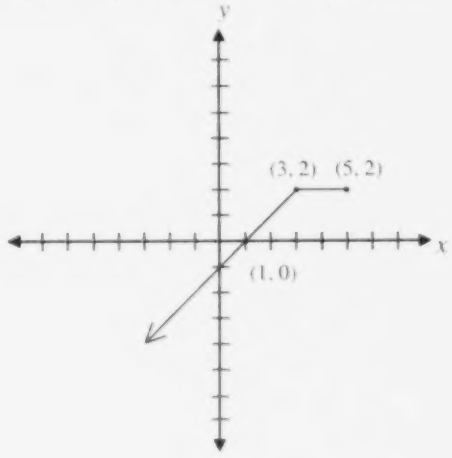
Part 2—Open-Response Questions

Question No. 41

Outcome: a) B6 b) B4

Question

41. The graph of a function $y = f(x)$ is shown below. Sketch the graphs of:

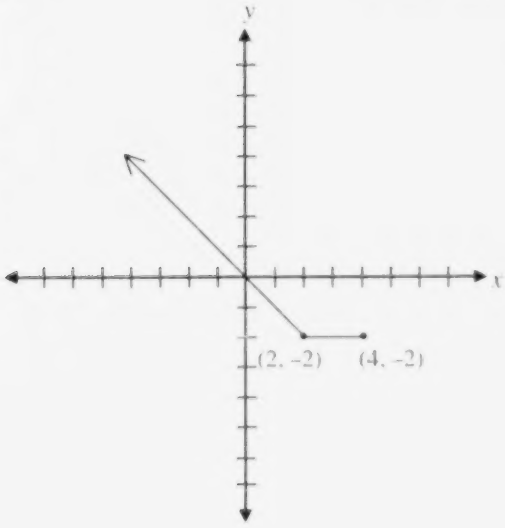


a) $y = -f(x+1)$ — 2 marks

b) $y = \dfrac{1}{f(x)}$ — 2 marks

Solution

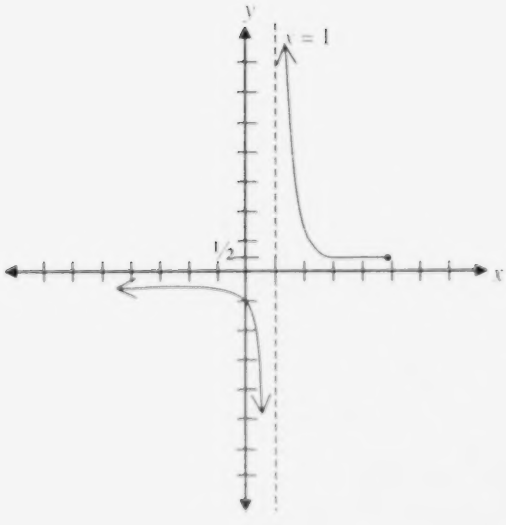
a)



1 mark for horizontal translation

1 mark for reflection in the x-axis

b)



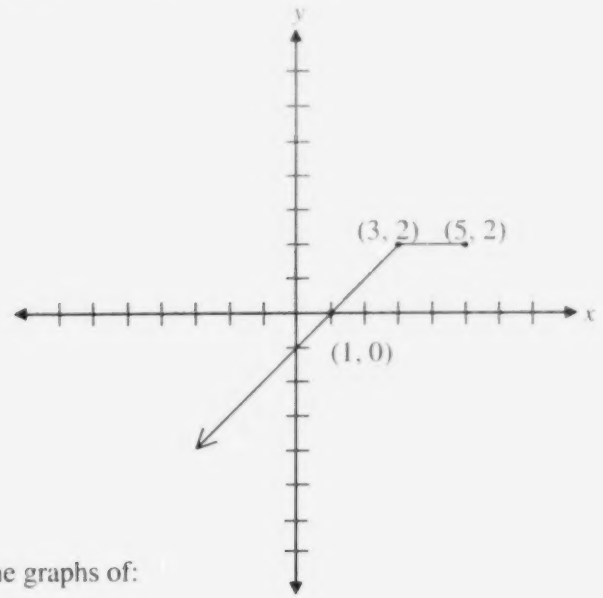
½ mark for correct asymptote

1 mark for graph (½ mark for each branch)

½ mark for accuracy

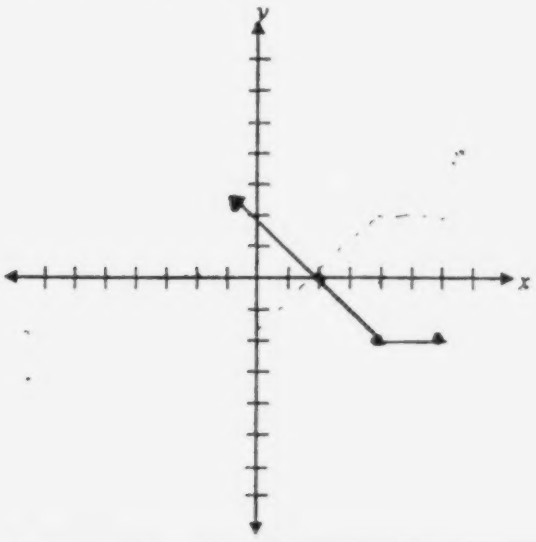
EXEMPLARS

41. The graph of a function $y = f(x)$ is shown below.

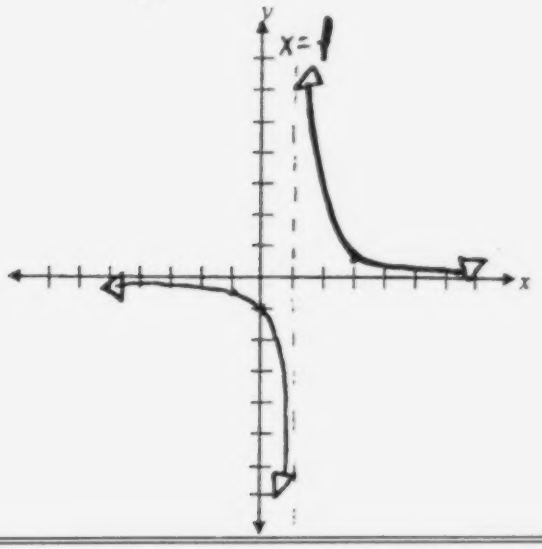


Sketch the graphs of:

a) $y = -f(x+1)$



b) $y = \frac{1}{f(x)}$



x	1	2		0	-1
y		½			

a) Mark: 1 out of 2
Note(s): • gave 1 mark for reflection in the x-axis

b) Mark: 1½ out of 2
Note(s): • deducted ½ mark for inaccuracy

Question No. 42

Outcome: A4

Question

42. Solve for θ in the interval $[\pi, 2\pi]$:

2 marks

$$(\tan\theta + 1)(2\cos\theta - 1) = 0$$

Solution

$\tan\theta = -1$; $\cos\theta = \frac{1}{2}$ 1 mark (½ mark for each function value)

$\theta = \frac{7\pi}{4}, \frac{5\pi}{3}$ 1 mark (½ mark for each answer)

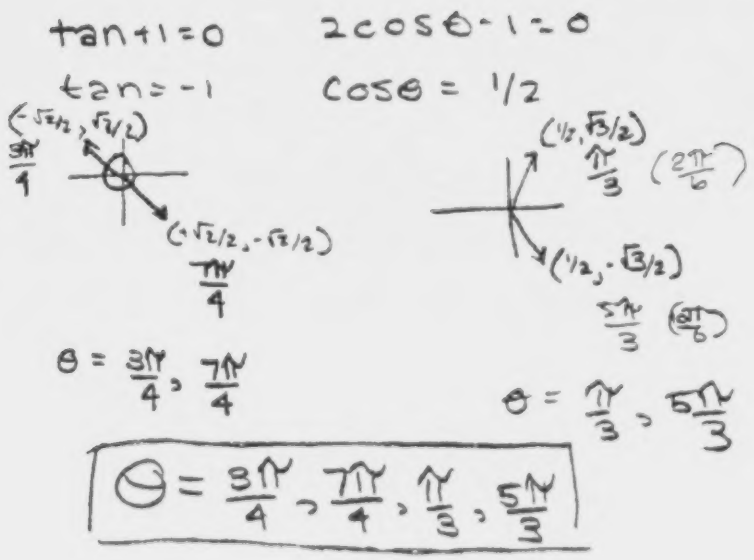
Note(s)

- **deduct** 1 mark for answers in degrees
- **deduct** ½ mark for correct answers in $[0, 2\pi]$

EXEMPLARS

42. Solve for θ in the interval $[\pi, 2\pi]$.

$$(\tan\theta + 1)(2\cos\theta - 1) = 0$$



Mark:	**1½ out of 2**
Note(s):	• deducted ½ mark for answers outside the given interval

Question No. 43

Outcome: C2, A3

Question

43. Find the exact value of:

a) $\sin\frac{11\pi}{12}$ — 3 marks

b) $\csc\frac{11\pi}{12}$ — 1 mark

Solution

a) $\frac{11\pi}{12} = \frac{2\pi}{3} + \frac{\pi}{4}$

$\sin\frac{11\pi}{12} = \sin\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$ ½ mark for correct combination

$= \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$ 2 marks (½ mark for each correct value)

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$ ½ mark for simplification

b) $\csc\frac{11\pi}{12} = \frac{4}{\sqrt{6}-\sqrt{2}}$ 1 mark for reciprocal

Note(s)

- other possible combinations: $\left(\frac{7\pi}{6} - \frac{\pi}{4}\right)$ or $\left(\frac{5\pi}{4} - \frac{\pi}{3}\right)$ or $(210° - 45°)$

EXEMPLARS

43. Find the exact value of:

a) $\sin\frac{11\pi}{12}$

$\sin\left(\frac{11\pi}{12}\right) = \sin\frac{2\pi}{3} + \sin\frac{\pi}{4}$

$= \sin\frac{2\pi}{3}\cos\frac{\pi}{4} + \cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \sin\left(\frac{\sqrt{3}}{2}\right)\cos\left(\frac{\sqrt{2}}{2}\right) + \cos\left(-\frac{1}{2}\right)\sin\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

Mark: **2 out of 3**
Note(s):
- deducted ½ mark for notation error in first line
- deducted ½ mark for notation error in third line

b) $\csc\frac{11\pi}{12}$

$\csc\frac{11\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$

Mark: **0 out of 1**

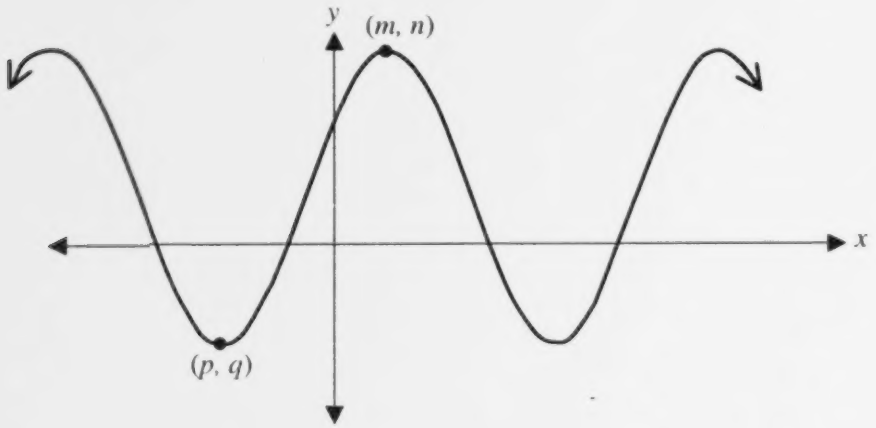
Question No. 44

Outcome: B7

Question

44. The graph of $y = 4\cos\left(x - \frac{\pi}{4}\right) + 2$ is sketched below.

3 marks



Find the exact values of m, n and p.

Solution

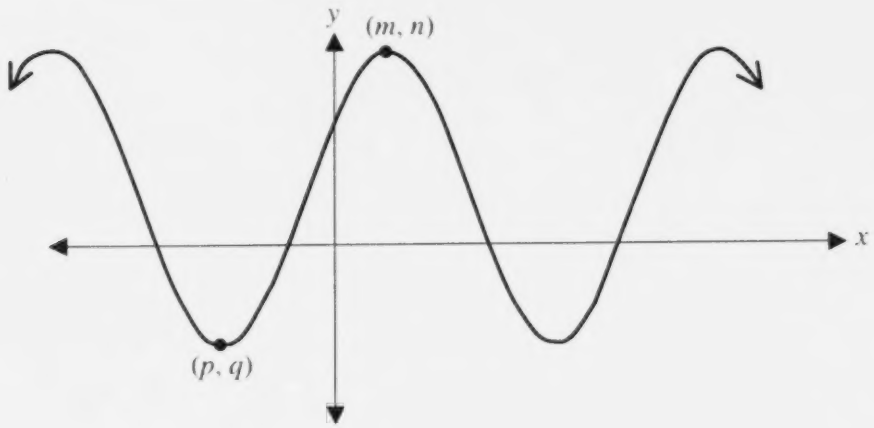
$m =$	$\frac{\pi}{4}$	1 mark
$n =$	6	1 mark
$p =$	$-\frac{3\pi}{4}$	1 mark

Note(s)

- if the value of m is wrong and the value of p is also wrong but consistent with m $(m - p = \pi)$, give 1 mark

EXEMPLARS

44. The graph of $y = 4\cos\left(x - \frac{\pi}{4}\right) + 2$ is sketched below.



Find the exact values of m, n and p.

$m =$ 6

$n =$ $\frac{\pi}{4}$

$p =$ -2

Mark: **0 out of 3**
Note(s): • all incorrect

Question No. 45

Outcome: C1

Question

45. Prove the following identity:

3 marks

$$\frac{\tan\theta \cdot \csc^2\theta}{\tan^2\theta + 1} = \cot\theta$$

Solution

$$\text{LHS} = \frac{\dfrac{\sin\theta}{\cos\theta} \cdot \dfrac{1}{\sin^2\theta}}{\sec^2\theta}$$

1½ marks (½ mark for each substitution)

$$= \frac{\dfrac{1}{\cos\theta\ \sin\theta}}{\dfrac{1}{\cos^2\theta}}$$

$$= \frac{1}{\cos\theta\ \sin\theta} \cdot \frac{\cos^2\theta}{1}$$

$$= \frac{\cos\theta}{\sin\theta}$$

1 mark for simplification

$$= \cot\theta = \text{RHS}$$

½ mark for substitution

EXEMPLARS

45. Prove the following identity:

$$\frac{\tan\theta \cdot \csc^2\theta}{\tan^2\theta + 1} = \cot\theta$$

LHS $\dfrac{\left(\frac{\sin}{\cos}\right)\left(\frac{1}{\sin^2}\right)}{\sec^2}$

$\dfrac{\frac{\sin}{\cos \sin^2} \cdot \frac{1}{\sec^2}}{\sec^2}$

$\dfrac{\sin}{\left(\frac{\cos}{1}\right)\left(\frac{\sin^2}{1}\right)\left(\frac{1}{\cos^2}\right)}$ $= \dfrac{\sin}{\frac{\sin^2}{\cos}} = \dfrac{\sin}{1} \cdot \dfrac{\cos}{\sin^2} = \dfrac{\sin\cos}{\sin^2}$ $= \dfrac{\cos}{\sin} = RHS$

$\dfrac{\cos\sin^2}{\cos^2}$

Mark:	**1 out of 3**
Note(s):	• gave 1½ marks for the substitutions
	• deducted ½ mark for missing θ's

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Question No. 46

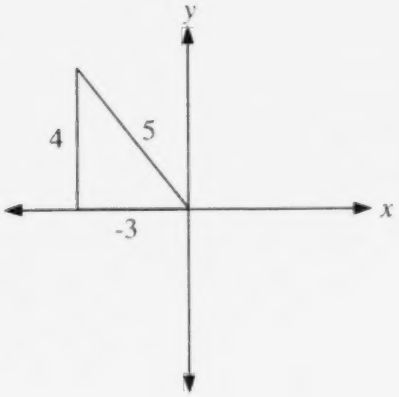
Outcome: C2, A3

Question

46. If $\tan\theta = -\frac{4}{3}$ and $\frac{\pi}{2} < \theta < \pi$, state the exact value of $\cos 2\theta$.

3 marks

Solution



1 mark for $r = 5$

Method 1

$\cos\theta = -\frac{3}{5}$ ½ mark for function

$\cos 2\theta = 2\cos^2\theta - 1$

$= 2\left(-\frac{3}{5}\right)^2 - 1$ ½ mark for substitution

$= 2\left(\frac{9}{25}\right) - 1$

$= \frac{18}{25} - 1$

$= -\frac{7}{25}$ 1 mark for answer

Question No. 46

Outcome: C2, A3

Question

46. If $\tan\theta = -\frac{4}{3}$ and $\frac{\pi}{2} < \theta < \pi$, state the exact value of $\cos 2\theta$.

Method 2

$\sin\theta = \frac{4}{5}$ — ½ mark for function

$\cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{4}{5}\right)^2$ — ½ mark for substitution

$= 1 - 2\left(\frac{16}{25}\right)$

$= 1 - \frac{32}{25}$

$= -\frac{7}{25}$ — 1 mark for answer

Method 3

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$

$= \frac{9}{25} - \frac{16}{25}$ — 1 mark { ½ mark for function values; ½ mark for substitution }

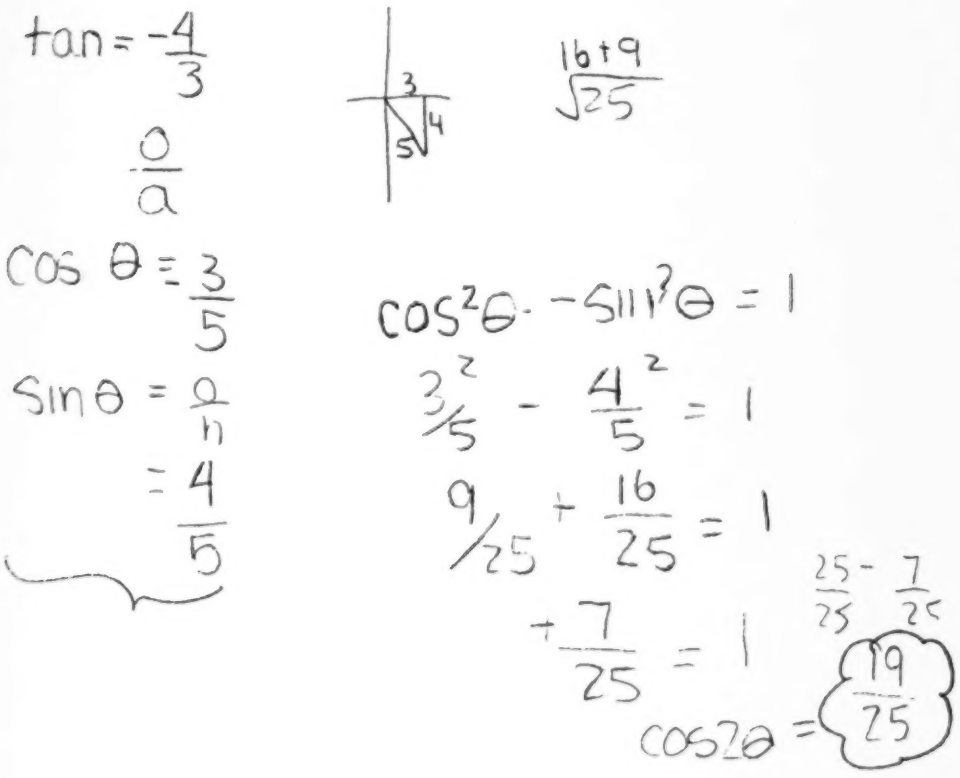
$= \frac{-7}{25}$ — 1 mark for answer

Note(s)

- give 1½ marks if student finds *r* and one function value correctly but proceeds incorrectly after that

EXEMPLARS

46. If $\tan\theta = -\frac{4}{3}$ and $\frac{\pi}{2} < \theta < \pi$, state the exact value of $\cos 2\theta$.



Mark:	**1½ out of 3**
Note(s):	• gave 1 mark for correct r value • gave ½ mark for correct value of $\sin\theta$ • $\cos^2\theta - \sin^2\theta = 1$ is incorrect formula

Question No. 47

Outcome: B3

Question

47. Show that $f(x) = x^3 - x$ is an odd function.

2 marks

Solution

Method 1

$f(x) = x^3 - x$

$f(-x) = (-x)^3 - (-x)$ ½ mark for substitution of $(-x)$

$= -x^3 + x$ ½ mark for simplification

$= -(x^3 - x)$ ½ mark for factoring

$= -f(x)$ ½ mark

$\therefore f(x)$ is odd

Method 2

$-f(-x) = -\left[(-x)^3 - (-x)\right]$ ½ mark for substitution

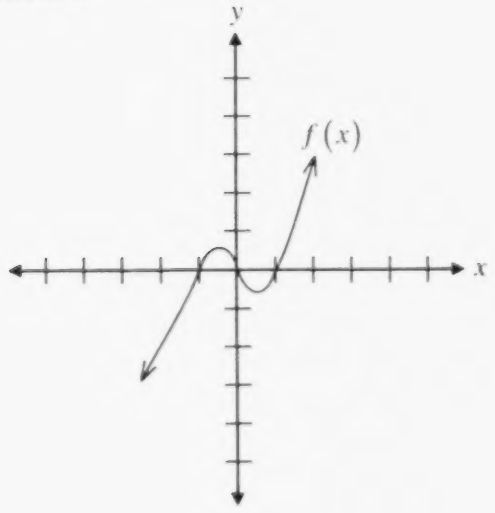
$= -\left[-x^3 + x\right]$

$= x^3 - x$ 1 mark for simplification

$= f(x)$ ½ mark

$\therefore f(x) = -f(-x)$

Method 3



1 mark for graph

1 mark for explanation

The graph of $f(x)$ is symmetrical in the origin

$\therefore f(x) = x^3 - x$ is an odd function

EXEMPLARS

47. Show that $f(x) = x^3 - x$ is an odd function.

$f(x) = x^3 - x$

$f(-x) = -x^3 + x$

Mark: **1 out of 2**

Note(s):
- gave ½ mark for substitution
- gave ½ mark for simplification

Question No. 48

Outcome: B5

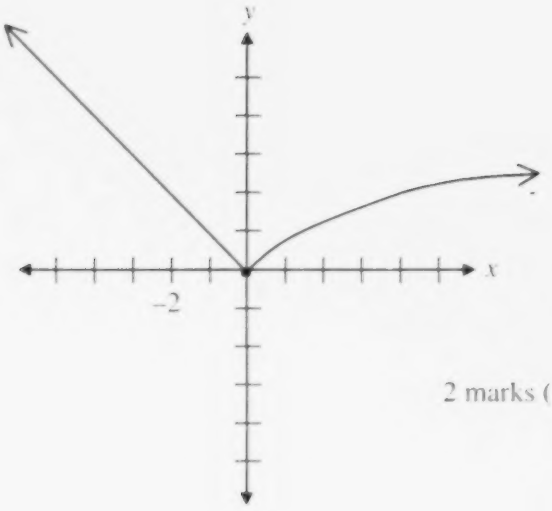
Question

48. Sketch the graph of $f(x)$ on the axes below.

2 marks

$$f(x)=\begin{cases}-x, & x<0\\ \sqrt{x}, & x\geq 0\end{cases}$$

Solution



2 marks (1 mark for each part of the graph)

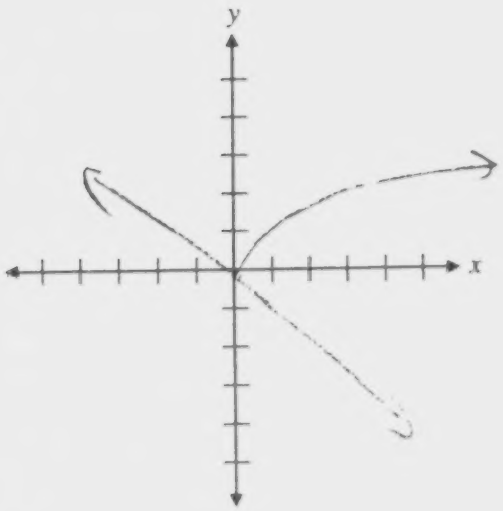
Note(s)

- **deduct** ½ mark for $x\geq 0$ if the graph is a straight line

EXEMPLARS

48. Sketch the graph of $f(x)$ on the axes below.

$$f(x)=\begin{cases}-x, & x<0\\ \sqrt{x}, & x\geq 0\end{cases}$$



Mark: 1½ out of 2

Note(s):
- gave 1 mark for $\sqrt{x}$
- gave ½ mark for $-x$

Question No. 49

Outcome: D6

Question

49. Solve for x. Give your answer in simplest form.

2 marks

$$\log_9(\log_4 x) = \frac{1}{2}$$

Solution

$\log_4 x = 9^{\frac{1}{2}}$	½ mark for exponential form
$\log_4 x = 3$	½ mark for simplification
$x = 4^3$	½ mark for exponential form
$x = 64$	½ mark for answer

EXEMPLARS

49. Solve for x. Give your answer in simplest form.

$$\log_9(\log_4 x) = \frac{1}{2}$$

$9^{\frac{1}{2}} = \log_4 X$

$\frac{\frac{1}{9}}{\log 4} = X$

Mark:	**½ out of 2**
Note(s):	• gave ½ mark for exponential form

Question No. 50

Outcome: H3

Question

50. Evaluate $\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{k}$.

2 marks

Solution

$$\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{k}$$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots$ ½ mark for t_1, ½ mark for r

$= \frac{\frac{1}{2}}{1-\frac{1}{2}}$ ½ mark for substitution

$= \frac{\frac{1}{2}}{\frac{1}{2}}$

$= 1$ ½ mark for answer

EXEMPLARS

50. Evaluate $\sum_{k=1}^{\infty}\left(\frac{1}{2}\right)^{k}$.

$\frac{1}{2}^1 + \frac{1}{2}^2 + \frac{1}{2}^3 + \frac{1}{2}^4$

$= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{32}$

$\frac{1}{6} + \frac{1}{8} = \frac{1}{14}$

$\frac{\frac{1}{2}}{1-\left(\times\frac{1}{2}\right)} = \frac{1}{4}$

$\frac{3}{2} \cdot \frac{1}{2} = \frac{1}{2}$

Mark:	**1 out of 2**
Note(s):	• gave ½ mark for t_1 • gave ½ mark for r • gave ½ mark for substitution • deducted ½ mark for arithmetic errors

Question No. 51

Outcome: D6

Question

51. Find the x-intercept(s) of the following function:

$$y = \log(10 - 3x) - 2\log x$$

4 marks

Solution

Method 1

$\log(10-3x) - 2\log x = 0$ — ½ mark for setting $y = 0$

$\log\left(\frac{10-3x}{x^2}\right) = 0$ — 1 mark for log theorems

$10^0 = \frac{10-3x}{x^2}$ — 1 mark for changing to exponential form

$x^2 = 10 - 3x$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = \cancel{-5}$ $\quad x = 2$ — 1 mark for solving for x

extraneous root — ½ mark for discarding the extraneous root

Method 2

$0 = \log(10-3x) - 2\log x$ — ½ mark for setting $y = 0$

$0 = \log(10-3x) - \log x^2$ — ½ mark for log theorem

$\log x^2 = \log(10-3x)$ — ½ mark

$x^2 = 10 - 3x$ — 1 mark for obtaining quadratic equation

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = \cancel{-5}$ $\quad x = 2$ — 1 mark for solving for x

extraneous root — ½ mark for discarding the extraneous root

Note(s)

- **deduct** 1 mark for incorrect use of anti-log

EXEMPLARS

51. Find the x-intercept(s) of the following function:

$$y = \log(10 - 3x) - 2\log x$$

$y = \log(10-3x) - \log x^2$

* both logs are on the same side and equal so they can be ignored!

$y = 10 - 3x - x^2$

$y = -x^2 - 3x + 10$ ← divide out a negative.

$y = -(x^2 + 3x - 10)$

$y = -(x+5)(x-2)$

$x + 5 = 0 \quad x - 2 = 0$

$x = -5 \quad x = 2$

$\boxed{x = -5, 2}$

Mark:	**1 out of 4**
Note(s):	• gave 1 mark for solving for x

Question No. 52

Outcome: E2

Question

52. Solve for n:

$${}_nP_2 = 12$$

2 marks

Solution

Method 1

$\frac{n!}{(n-2)!} = 12$ ½ mark for setup of formula

$n(n-1) = 12$ ½ mark for simplification

$n^2 - n - 12 = 0$

$(n-4)(n+3) = 0$

$n = 4$ ~~$n = -3$~~

$\therefore n = 4$ 1 mark for answer

Method 2

$n(n-1) = 12$ 1 mark for setup of consecutive numbers

$\therefore n = 4$ 1 mark for answer

Note(s)

- **deduct** ½ mark if n = – 3 is not discarded

EXEMPLARS

52. Solve for n:

$${}_nP_2 = 12$$

$6 \cdot 5$
30
$5 \cdot 4$
20
$4 \cdot 3$
12

$\boxed{n = 4}$

Mark:	**2 out of 2**
Note(s):	• gave 1 mark for trial and error method • gave 1 mark for correct answer

Question No. 53

Outcome: A2, A4

Question

53. Solve for x over $[0, 2\pi]$:

3 marks

$$\cos 2\theta = \frac{\sqrt{3}}{2}$$

Solution

$$\cos 2\theta = \frac{\sqrt{3}}{2}$$

$2\theta = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$ — 2 marks (½ mark for each value)

$\theta = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$ — 1 mark for dividing by 2

EXEMPLARS

53. Solve for x over $[0, 2\pi]$:

$$\cos 2\theta = \frac{\sqrt{3}}{2}$$

$\cos(2\theta) = \frac{\sqrt{3}}{2}$

ref. angle $= \frac{\pi}{3}$

$\cos(2\theta) = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{9\pi}{3}$

$\theta = \frac{\pi}{6}, \frac{2\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}$

Mark: **1 out of 3**
Note(s): • gave 1 mark for dividing by 2

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